AL.8.8063-81
NO ILL
V.2
University of Alberta Library
0 1620 3410397 6

Module 2

MATHEMATICS 10

Patterns and Relations

Learning Technologies Branch

Alberta LEARNING

Digitized by the Internet Archive
in 2016 with funding from
University of Alberta Libraries

https://archive.org/details/puremathematics102albe

Pure MATHEMATICS 10

Module 2
Patterns and Relations

Learning
Technologies
Branch

Alberta
LEARNING

Pure Mathematics 10
Module 2: Patterns and Relations
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2139-4

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

ALL RIGHTS RESERVED

Copyright © 2002, the Crown in Right of Alberta, as represented by the Minister of Learning, Alberta Learning, 11160 Jasper Avenue, Edmonton, Alberta T5K 0L2. All rights reserved. Additional copies may be obtained from the Learning Resources Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Learning.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Learning so that appropriate corrective action can be taken.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

Welcome

Welcome to Module 2. We hope you'll enjoy your study of Patterns and Relations.

Pure Mathematics 10

Module 1
Statistics and Data Tables

Module 2
Patterns and Relations

Module 3
Algebra

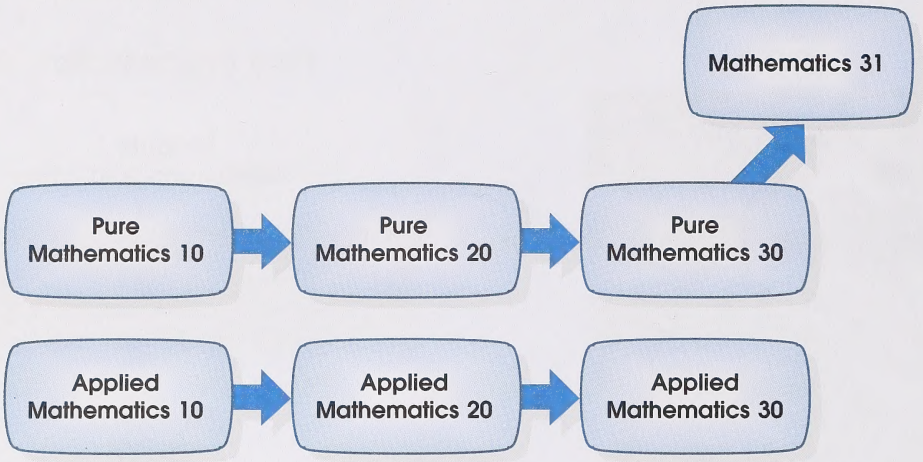
Module 4
Number Connections

Module 5
Trigonometry

Pure Mathematics 10 contains five modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

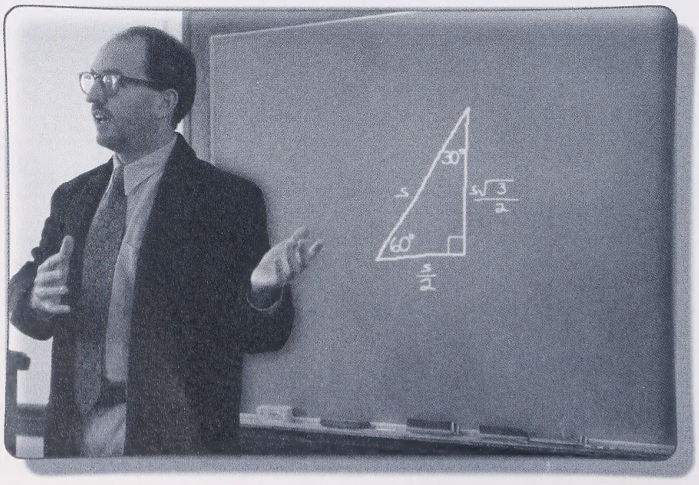
Pure Mathematics 10 is the first course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.

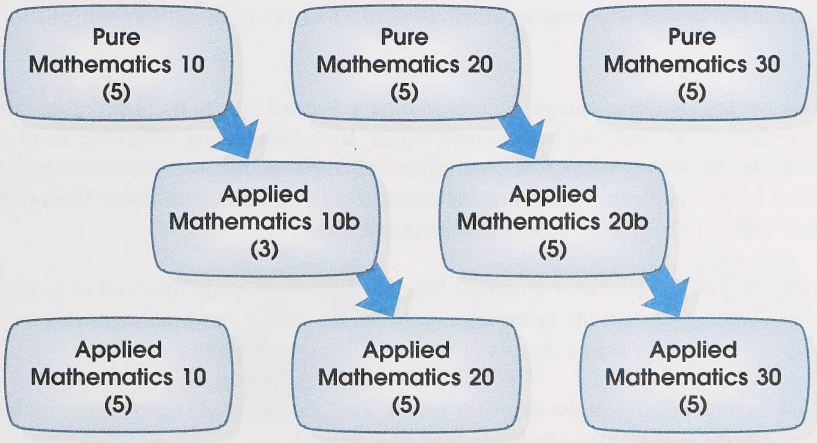


Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

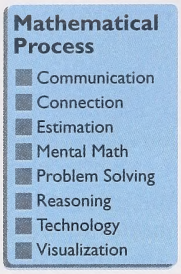
If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you will take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Applied Mathematics 10b (3)
Applied Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems



This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an "aha!") or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 10 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Centre:

- the *MATHPOWER™ 10* textbook and data disks, Western Edition, published by McGraw-Hill Ryerson (1998)

- a graphing calculator
  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* Algebra FX-2.0, CFX-9850 GA-Plus, CFX-9850 G, CFX-9800 G, or FX-9700 series; *Sharp* EL-9600C, EL-9600, EL-9200, or EL-9300; or *Hewlett-Packard* HP 39g (only approved for 2001–2003 school years). **Note:** Some calculators are no longer commercially manufactured.

- spreadsheet program
  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh or for Windows, *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable.

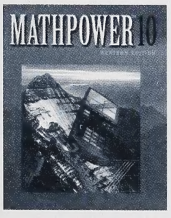
You will need access to a computer (mandatory) and a videocassette player (optional).

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.

View a videocassette.

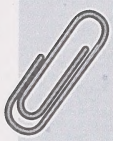
Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.

Use the Computer Data Bank.

Remember: *Any Internet website address given in this module is subject to change.*

CONTENTS



Module Overview

Evaluation
2
Strategies for Completing a Module
3

Section 1

Coordinate Geometry

Activity 1: Distance and Midpoint Formulas
6
Activity 2: Slope
14
Activity 3: Linear Equations
17
Activity 4: Parallel and Perpendicular
27
Activity 5: Graphing Linear Equations
29

Follow-up Activities
34
Extra Help
34
Enrichment
37

Conclusion
41
Assignment
41

Section 2

Relations and Functions

Activity 1: Relations
44
Activity 2: Linear Relations
46
Activity 3: Non-Linear and General Relations
55
Activity 4: Functions
58
Activity 5: Linear Functions
61

Follow-up Activities
65
Extra Help
65
Enrichment
67

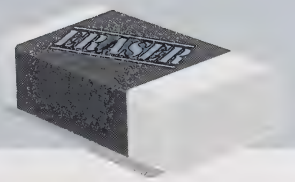
Conclusion
71
Assignment
71

Module Summary

Final Module Assignment
72

Appendix

Glossary
74
Suggested Answers
75

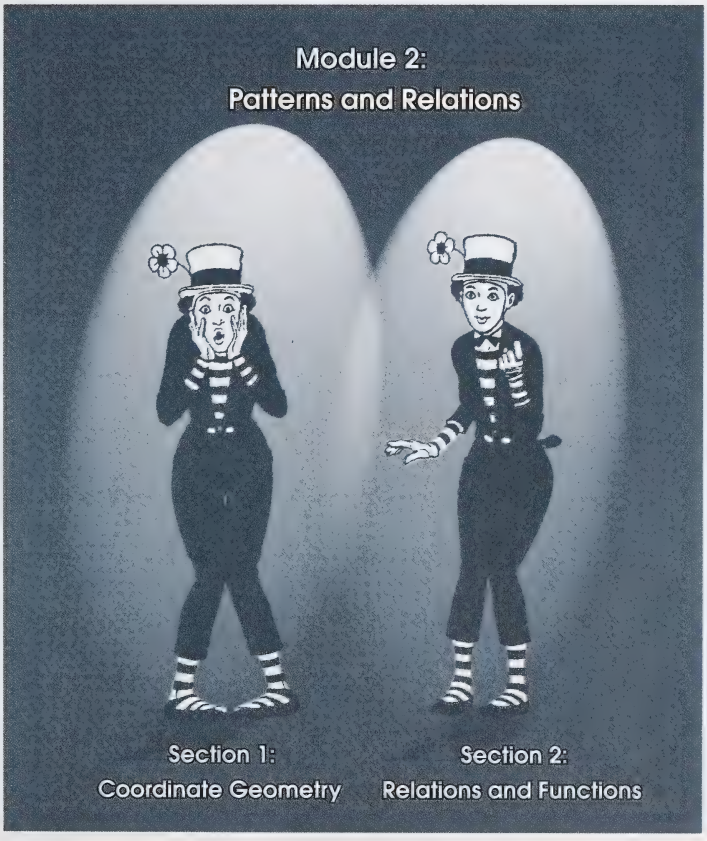


Module Overview

It is remarkable how easily a mime, without the use of props or speech, can communicate to an audience. Through facial expressions and the skilled use of hands and body, a mime's performance can be appreciated by young and old. A mime's performance relies on a symbolic representation of emotion and the physical world, giving meaning to the performance.

Like a mime, a mathematician uses symbols to describe relationships, then paints those relations as graphs to aid your understanding—to help you appreciate the underlying principles.

In this module, you will be introduced to coordinate geometry—a visual representation of relationships. Then, you will investigate these relationships further as relations and functions.



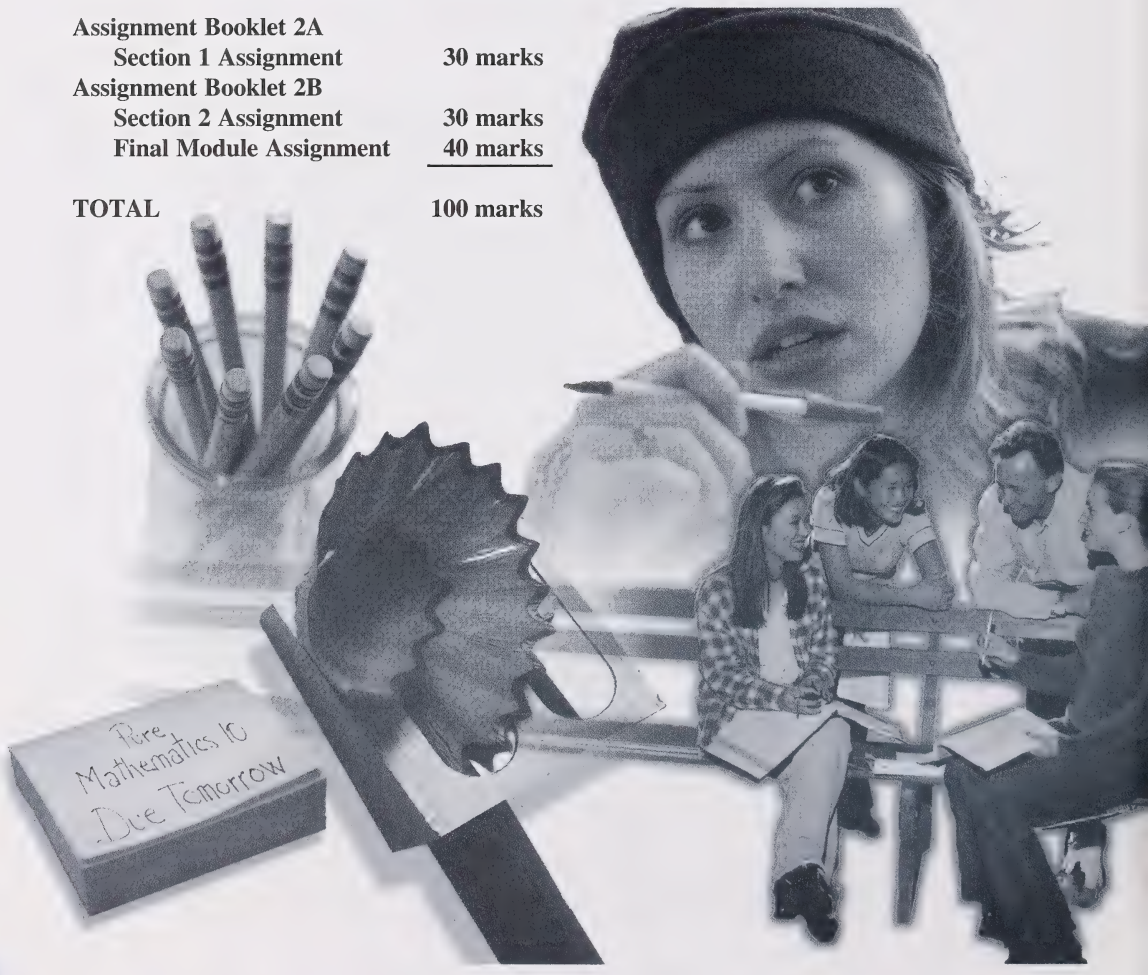
Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Patterns and Relations, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

In this module, you will be directed to the accompanying Assignment Booklets. You are expected to complete two Assignment Booklets for this module. Your grading in this module is based in part upon the assignments you submit for evaluation. The mark distribution is as follows:

Assignment Booklet 2A	
Section 1 Assignment	**30 marks**
Assignment Booklet 2B	
Section 2 Assignment	**30 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**



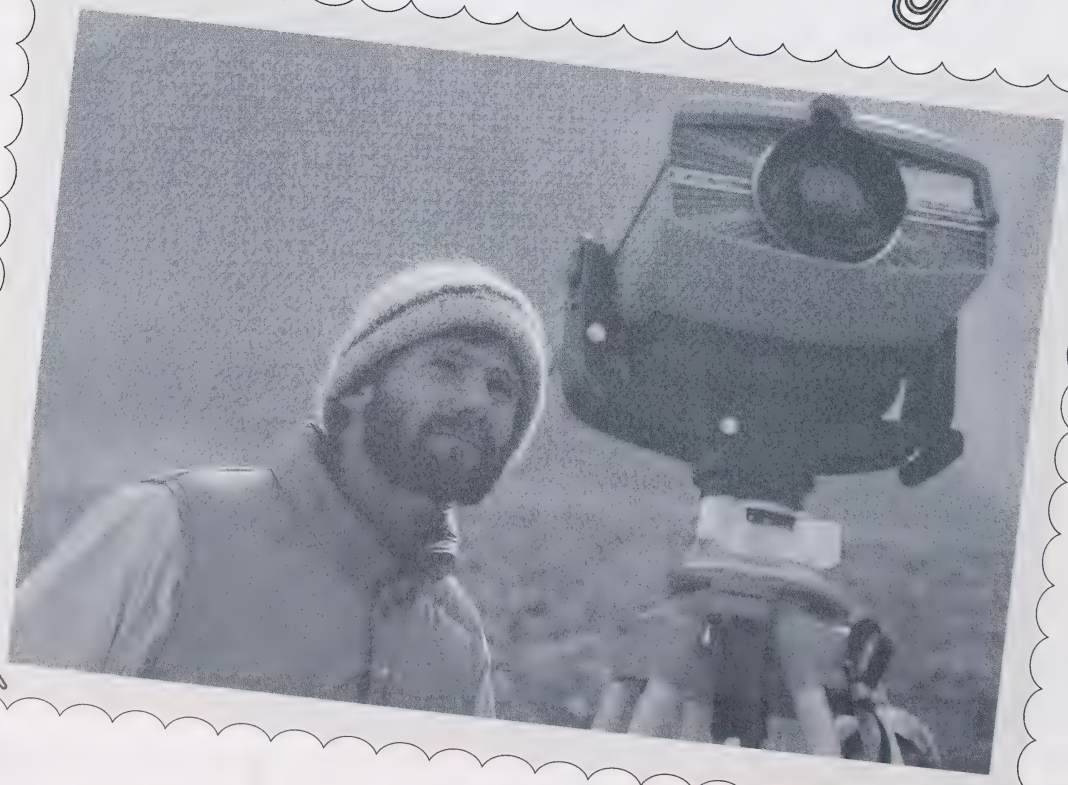
Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 10.

Good luck!

Section 1



Coordinate Geometry

Mapmakers rely on surveyors to take careful measurements. These measurements are used to draw maps that communicate the precise positions of the features of the physical world—features such as hills, rivers, roads, and buildings. Planners and developers reading these maps make decisions based on the relationships among the features.

Mathematicians use the coordinate plane to position points, lines, triangles, and other geometric figures. The tools of algebra may be used to describe and analyse each figure's features and relationships.

In this section, you will investigate points, the distance between them, and the segments and lines defined by points. You will solve problems involving midpoints of segments, the slopes of lines and segments, the equations of lines, and the relationships among parallel and perpendicular lines.

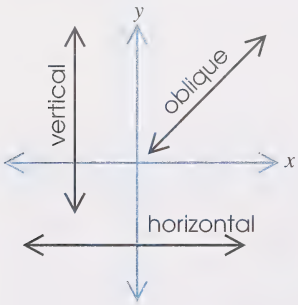
Activity 1: Distance and Midpoint Formulas

Distance Between Two Points

Aircraft appear as points of light on a radar screen. Radar is not only used to determine the location of each aircraft relative to the radar station, it is also used to determine each aircraft's position relative to other aircraft.

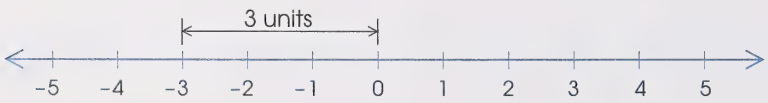
The coordinate plane enables you to find the distance between any two points. You may recall that two distinct points, when joined, determine one of the following: a horizontal, vertical, or **oblique** segment.

oblique: neither horizontal nor vertical



In order to develop and work with the formulas for the lengths of segments, you need to understand and use **absolute value**. Absolute value answers the question, "How far does a given real number lie from zero on the real number line?" Because you are only interested in how far the number is from zero and not whether it lies on the left or right of zero, the absolute value of any real number, x, (written $|x|$) is non-negative. For example, $|-3| = 3$, as shown in the diagram.

absolute value: the distance a real number, x, lies from the origin on the number line, denoted by $|x|$



Example

State the absolute value of each of the following real numbers:

a. -5 **b.** 6 **c.** 0 **d.** $x \geq 0$ **e.** $x < 0$

Solution

a. $|-5| = 5$ **b.** $|6| = 6$ **c.** $|0| = 0$ **d.** $|x| = x$ **e.** $|x| = -x$

Notice that the absolute value of a non-negative real number is that number itself, but the absolute value of a negative number is its opposite! You could find $|-5|$ as follows:

$$|-5| = -(-5) = 5$$

opposite of (-5)

Also, absolute value is treated like parentheses when you decide on the order of operations in an expression.

Example

Evaluate each of the following:

a. $|-3-8|$ **b.** $|-3|-|-8|$

Solution

a. As with parentheses, evaluate within the absolute value sign first.

$$\begin{aligned} |-3-8| &= |-11| \\ &= 11 \end{aligned}$$

b. This time, find the absolute values before subtracting.

$$\begin{aligned} |-3|-|8| &= 3-8 \\ &= -5 \end{aligned}$$

Mathematical Process

- [ ] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [ ] Visualization

Turn to page 7 of *MATHPOWER™ 10* and work through Example 3.

1. Answer the following questions on pages 8 and 9 of the textbook:

a. questions 31 to 39 of "Practice"
b. questions 66.c. and 66.f. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

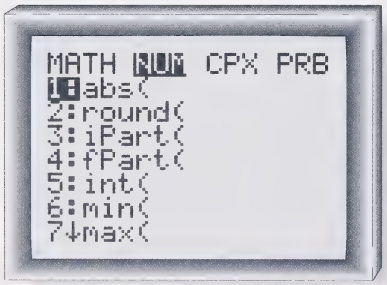
You can use your graphing calculator to evaluate expressions involving absolute value.

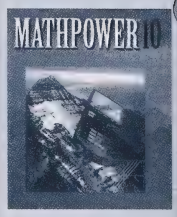
Example

Use your graphing calculator to evaluate $\left|-2-(-5)\right|$.

Solution

Step 1: To access the absolute value function, press (MATH). Then, use the arrow keys to select the NUM menu. If, in the NUM menu, absolute value is highlighted, as shown, press (ENTER). If it is not highlighted, press (1).



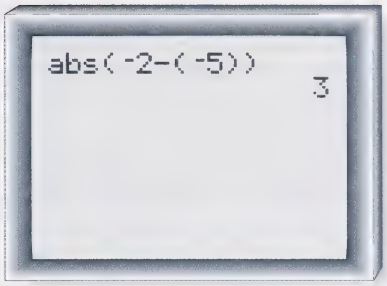


Mathematical Process

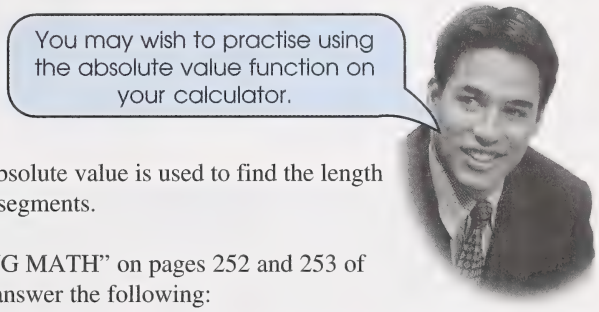
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [X] Technology
- Visualization

Step 2: Now, press the following sequence of keys:

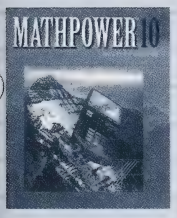
$\therefore \ |-2-(-5)| = 3$



Step 3: Check your answer using pencil and paper.



Next, you will discover how absolute value is used to find the length of vertical and horizontal line segments.

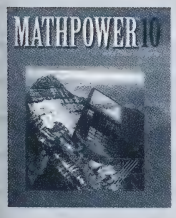


2. Turn to "INVESTIGATING MATH" on pages 252 and 253 of *MATHPOWER™ 10* and answer the following:

a. questions 1 to 5 of Investigation 1, "Lengths on a Number Line"
b. questions 1 to 4 of Investigation 2, "Lengths of Horizontal Line Segments"
c. questions 1 to 4 of Investigation 3, "Lengths of Vertical Line Segments"
d. questions 1.a., 1.c., and 2 of Investigation 4, "Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

You will now use the results from your investigation of horizontal and vertical segments to develop and apply a formula that will give you the length of any segment: vertical, horizontal, or oblique.



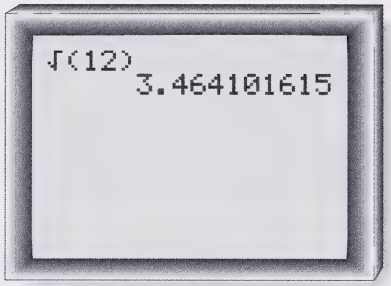
3. Turn to pages 254 and 255 of *MATHPOWER™ 10* and answer the following:

a. questions a. to d. of "Explore: Develop a Method"
b. questions 1 to 6 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 1.

When determining the lengths of oblique segments, as part of the process, you must find square roots. Suppose you had to find the square root of a positive number that is not a perfect square, such as 12. You can approximate the square root using your graphing calculator.

2nd [√] 1 2) ENTER



If you were asked to express your answer to the nearest tenth, you would write $\sqrt{12} \doteq 3.5$.

In many situations, you are expected to give the exact answer. You could write the square root of 12 as $\sqrt{12}$, or you could simplify it as $2\sqrt{3}$. To see why $\sqrt{12} = 2\sqrt{3}$, work through the following example.

Example

Simplify each of the following:

a. $\sqrt{12}$ **b.** $\sqrt{18}$ **c.** $\sqrt{50}$

Solution

In each instance, look for the largest perfect square that is a factor of the number appearing under the square root sign.

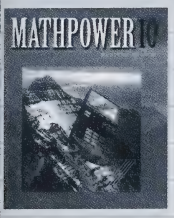
a. $\sqrt{12} = \sqrt{4 \times 3}$
$= \sqrt{4} \times \sqrt{3}$
$= 2\sqrt{3}$

b. $\sqrt{18} = \sqrt{9 \times 2}$
$= \sqrt{9} \times \sqrt{2}$
$= 3\sqrt{2}$

c. $\sqrt{125} = \sqrt{25 \times 5}$
$= \sqrt{25} \times \sqrt{5}$
$= 5\sqrt{5}$

You will examine simplifying square roots in Module 4 of this course.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization



Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization

You will now practise using the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find lengths of line segments. You will also apply this formula in problem-solving situations.

Turn to pages 255 and 256 of *MATHPOWER™ 10* and work through Examples 1 and 2.

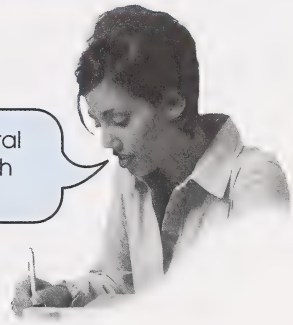
4. Answer the following questions on pages 256 and 257 of the textbook:

a. questions 1 to 6 of "Practice"
b. questions 13.a., 14, 20.a., and 24 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

You should now be comfortable using the formulas for the lengths of vertical, horizontal, and oblique segments.

Horizontal	Vertical	Oblique
$\lvert x_2 - x_1 \rvert$	$\lvert y_2 - y_1 \rvert$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$


The formula for oblique segments is a general distance formula that may be used for both vertical and horizontal segments.

5. Use the general distance formula and the formula for either vertical or horizontal segments to answer each of the following:

a. Find the length of the segment joining $A(1, 5)$ and $B(7, 5)$.

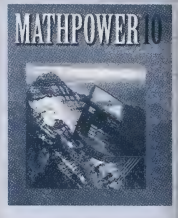
b. Find the length of the segment joining $A(1, 5)$ and $B(1, -3)$.

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now Try This

The driving or walking distance between two points in a city often differs from the direct distance between the points. How would you determine the shortest walking or driving distance?

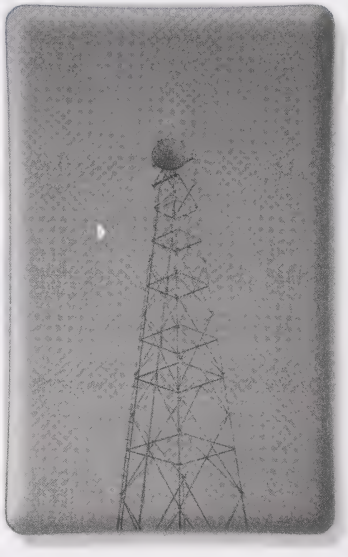
6. Turn to "GETTING STARTED" on page 250 of *MATHPOWER™ 10* and read through the introduction to Investigation 1, "Taxicab Geometry"; then answer questions 1.a., 2, and 3.



Compare your responses with those in the Appendix, Section 1: Activity 1.

Midpoint of a Line Segment

Notice how the photographer has captured the symmetry in the design of the microwave tower. The two nearest sides shown meet in a line that appears to split the tower in half. The diagonal braces along each side cross at their midpoints. Do you see any other examples in the photograph of structures that are divided in two?



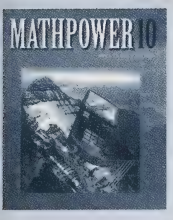
In coordinate geometry, the ability to locate the midpoint of a line segment is a fundamental skill.

7. Turn to "INVESTIGATING MATH" on page 258 of *MATHPOWER™ 10* and answer the following:

a. questions 1.a. to 1.c., 2, 3.a., 3.c., and 4 of Investigation 1, "Midpoints of Horizontal Line Segments"

b. questions 1.a. to 1.c., 2, 3.a., 3.c., and 4 of Investigation 2, "Midpoints of Vertical Line Segments"

Compare your responses with those in the Appendix, Section 1: Activity 1.



In the first part of this section, you used the results for horizontal and vertical segments to obtain a general distance formula that was applicable to all segments. In this part of the section, you again will use the results for horizontal and vertical segments to obtain a general midpoint formula.

Turn to page 259 of *MATHPOWER™ 10* and read "Midpoint of a Line Segment."

8. Answer the following questions on page 259 of the textbook:

a. questions a. to c. of "Explore: Use the Diagrams"
b. questions 1 to 5 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

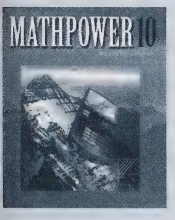
Make sure when you are using the midpoint formula, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, that you use brackets to indicate that your answer is an ordered pair.

Turn to page 260 of *MATHPOWER™ 10* and work through Examples 1 and 2.

9. Answer the following questions on pages 261 and 262 of the textbook:

a. questions 1, 3, 5, 8, 11, and 12 of "Practice"
b. questions 21, 22, 25.d., 29, and 36.a. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.



Mathematical Process
- [ ] Communication
- [ ] Connection
- [ ] Estimation
- [x] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Now Try This

The ability to evaluate expressions of the form $\frac{a+b}{2}$ mentally is a useful skill when working with the midpoint formula.

10. Turn to page 251 of *MATHPOWER™ 10* and answer questions 1 to 6 of "Mental Math: Order of Operations."

Compare your responses with those in the Appendix, Section 1: Activity 1.

Looking Back

In this activity, you solved problems involving distances between points in the coordinate plane. You also solved problems involving midpoints of line segments.

Suppose a friend telephoned you asking for help in finding the midpoint of the line segment joining $A(1, 6)$ and $B(-3, -2)$. What would you say? Write a possible conversation in your journal.

Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

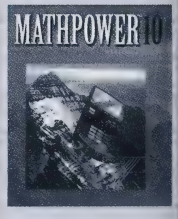
Activity 2: Slope

In those parts of the country where snowfall is high, homes are often built with steeper roofs to reduce accumulations. The pitch or slope of a roof is described by a ratio. On the prairies, where snow accumulations are less of a problem, a commonly used pitch in home construction is 4 in 12. In the imperial system of measure, a slope of 4 in 12 means the roof rises 4 inches for a horizontal run of 12 inches (1 foot). A slope of 4 in 12 can be written as 4:12 or 1:3. In the metric system, this ratio tells you the roof rises 10 cm for a horizontal run of 30 cm.

The terms **rise** and **run** are also used in coordinate geometry to describe the **slope** of a line.

Mathematical Process
- [ ] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

rise: the vertical displacement between two points



run: the horizontal displacement between two points

slope: the ratio of rise to run that measures the steepness of a line

Turn to page 263 of *MATHPOWER™ 10* and read "Slope."

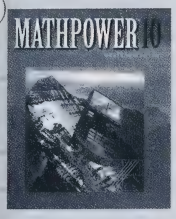
1. Answer the following questions on page 264 of the textbook:

a. questions a. to c. of "Explore: Develop a Method"
b. questions 1 to 6 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Your next task will be to explore the formula for the slope of a line using the coordinates of any two points on the line. Pay particular attention to how the slope of a line changes depending on its orientation in the coordinate plane.

Turn to pages 264 to 266 of *MATHPOWER™ 10* and work through Examples 1 to 5.



Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

2. Answer the following questions on pages 267 and 268 of the textbook:

a. questions 1 to 4, 8, 10, 12, and 16 of "Practice"

b. questions 30.a., 30.e., 30.f., 32.e., and 32.f. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

You may wish to refer to the chart on the right when solving problems involving slope.

Line	Graph	Slope
rises		+
horizontal		0
falls		-
vertical		undefined

When using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope of a non-vertical segment from its endpoints, it doesn't matter which of the two endpoints you use for (x_1, y_1) as long as you call the other (x_2, y_2).

Example

Find the slope of AB given $A(1,2)$ and $B(5,4)$.

Solution

Method 1

Let $(x_1, y_1) = (1,2)$

and $(x_2, y_2) = (5,4)$.

$$\therefore m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-2}{5-1}$$
$$= \frac{2}{4}$$
$$= 0.5$$

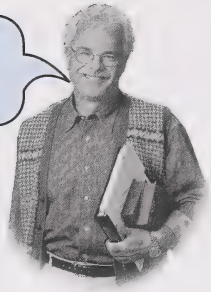
Method 2

Let $(x_1, y_1) = (5,4)$

and $(x_2, y_2) = (1,2)$.

$$\therefore m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2-4}{1-5}$$
$$= \frac{-2}{-4}$$
$$= 0.5$$

Because it doesn't matter which point is given first, you could also write the slope formula as $m = \frac{y_1 - y_2}{x_1 - x_2}$.



Turn to page 267 of *MATHPOWER™ 10* and work through Example 6.

3. Answer the following questions on pages 268 and 269 of the textbook:

a. questions 24 and 25 of "Practice"
b. questions 42.a. and 42.b. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

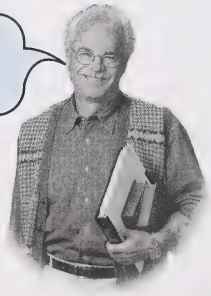
In the sciences, the slopes of graphs are often interpreted as rates. Speeds (km/h), wage rates ($/h), cooling rates (°C/s), and expansion rates (m/°C) are but a few.

Turn to page 270 of *MATHPOWER™ 10* and read "Slope as Rate of Change."

4. Answer the following questions on page 270 of the textbook:

a. "Explore: Interpret the Graph"
b. questions 1 to 3 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 2.

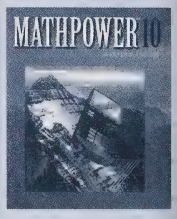


Turn to pages 270 and 271 of *MATHPOWER™ 10* and work through "Example: Internet Access."

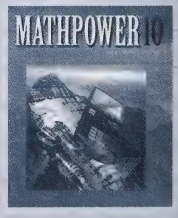
5. Do questions 1, 5, and 6 of "Applications and Problem Solving" on pages 271 and 272 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

Now Try This

Developing logic to solve problems is a critical skill in mathematics. For many years people have enjoyed solving logic problems. In fact, some of the problems still around today were posed originally by the ancient Egyptians.

6. Turn to page 269 of *MATHPOWER™ 10* and answer "Logic Power."

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Looking Back

In this activity, you solved problems involving rise and run and the slope of line segments.

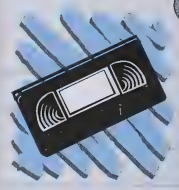
Suppose you were helping a friend prepare for a test on coordinate geometry. Make up a possible question that your friend would have to answer using the slope, distance, and midpoint formulas. Write that question and a possible solution in your journal.

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

Activity 3: Linear Equations

The trail of smoke from the launch of the shuttle traces out the trajectory of the booster rockets that blast the craft into orbit. The trajectory is planned precisely and is confirmed by the computers in mission control. Mathematical equations are an integral part of this process. In this activity, you will find the equation of a line from its location in the plane and locate a line from its equation.

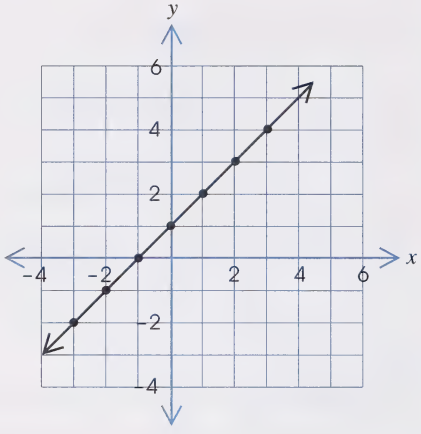
To assist you in your exploration, you will be using your graphing calculator. If you are using a TI-83 Graphing Calculator and have access to the video *The TI-83 Graphing Calculator Video Tutor*, you should view Section 1. This section of the video introduces the TI-83 graphing calculator, discusses graphing linear equations, and introduces the zoom and table features. Contact your school media centre or ACCESS, or contact the Learning Resources Centre, to check on the availability of this useful video.



Example

Use the graph on the right to answer the following:

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4



a. Explain why the points in the table are **collinear points**; that is, explain why the points lie in a straight line.

b. For each ordered pair, how is the y-coordinate related to the x-coordinate?

c. What is the equation of the line?

d. Graph your equation on a graphing calculator.

collinear points: points that lie in the same straight line

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

Solution

a. The graph reveals that the slope determined by any two points is 1. For instance, the slope of the segment joining $(-3,-2)$ and $(-2,-1)$ is 1, because the rise and run are both 1. The slope will be the same for any two successive points in the table of values. It is like walking up a set of stairs. Because the slope is constant, the points are collinear.

b. For each ordered pair, the y-coordinate = x-coordinate $+1$. For instance, for the ordered pair $(3,4)$, $4 = 3+1$.

Note: Check the ordered pairs for the other points.

c. The equation of the line describes how the y-coordinate is related to the x-coordinate.

$$y\text{-coordinate} = x\text{-coordinate} + 1$$
$$y = x + 1$$

Note: Because this is the equation of a straight line, it is called a **linear equation**.

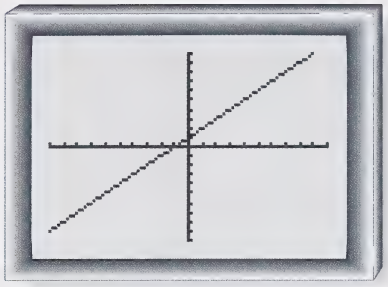
linear equation: the equation of a line, where each term in the equation is either a constant or of degree 1

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

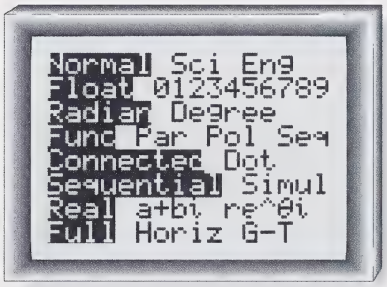
d. If you are graphing this linear equation on a TI-83 graphing calculator, use these keystrokes:

Y= X,T,Θ,*n* + 1 GRAPH



Remember: To obtain a graph similar to the one shown in this problem, press ZOOM 6.

If you have not used the graphing features of your calculator, begin by checking the mode settings. Press MODE to display the settings. You will use the factory settings (as shown in the display).



If your display differs from the factory settings, use the arrow keys to move the cursor to the desired setting and press ENTER. You should also check the factory settings in the format menu. Press 2nd [FORMAT]. You should see the following display:

Again, if your display differs from the factory settings, use the arrow keys to move the cursor to the desired setting and press ENTER.

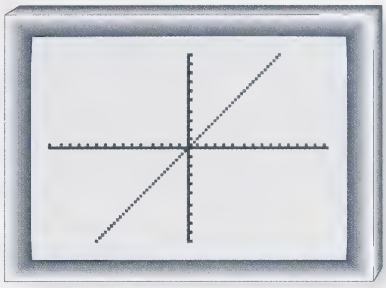
Next, to move between a graph and a table of values similar to the tables in question 1, press 2nd [TBLSET]. Change the settings to those shown in the following display:

These settings start the table of values at $x = -3$ and shows successive values that differ by 1; that is, -3, -2, -1, and so on. The y-coordinates are calculated automatically.

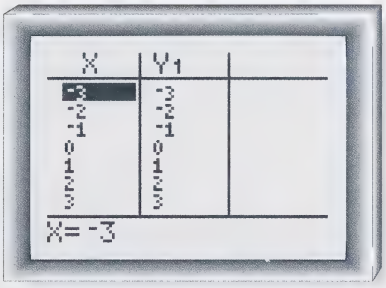
In order to have linear graphs in the viewing window resemble those obtained on graph paper, press ZOOM. You will see the following menu:

Select "6:ZStandard" by pressing 6. This graphs the line on a standard set of axes. Then press ZOOM and select "5:ZSquare" by pressing 5. This squares up the viewing window.

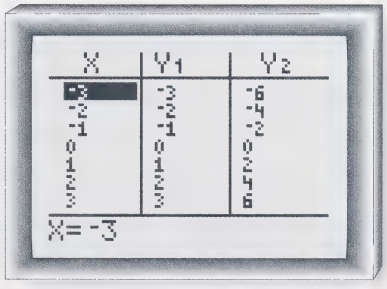
The graph $y = x$ should cross the x-axis at 45°, splitting the first quadrant into halves. Check this by pressing [Y=] [X,T,Θ,n] [GRAPH].



To see the table of values, press [2nd] [TABLE].



To enter a second linear equation, press [Y=]. You will see a flashing cursor on "$Y_1 = X$." You may press [CLEAR] to remove the original equation and enter a new equation. Or, if you wish to keep the first equation and enter a second equation so that two lines appear on the graph, press [ENTER]. This keystroke moves you down to "$Y_2 =$." Enter your second equation now. When you access a table of values, the display will show a Y_1 column for the first line and a Y_2 column for the second. In this example, $y_2 = 2x$.



1. Using graph paper, draw each line using the given partial tables; then determine the equation of each line. Graph your equation on a graphing calculator to check your answers.

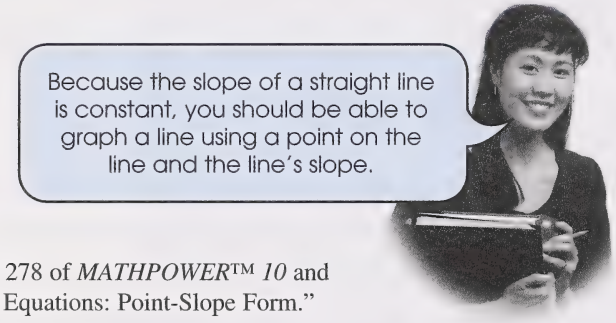
a.

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

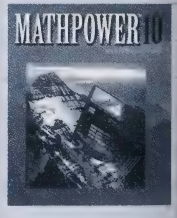
b.

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1

Compare your responses with those in the Appendix, Section 1: Activity 3.


Because the slope of a straight line is constant, you should be able to graph a line using a point on the line and the line's slope.

Turn to page 278 of *MATHPOWER™ 10* and read "Linear Equations: Point-Slope Form."



2. Answer the following questions on page 278 of the textbook:

a. questions a. and b. of "Explore: Develop a Method"
b. questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 3.

point - slope form: the form of an equation of a line based on the slope and a point on the line

You will now develop and apply in problem situations the **point-slope form** of a linear equation. This formula allows you, as its name suggests, to find the equation of a line from a point on the line and the slope of the line.

Turn to page 278 of *MATHPOWER™ 10* and read from the red line to Example 1 on page 279.

3. Express each of the following linear equations in **standard form** $(Ax + By + C = 0)$:

a. $y = \frac{2}{3}x + 4$ **b.** $y - 2 = \frac{3}{5}x - 7$

Compare your responses with those in the Appendix, Section 1: Activity 3.

Turn to pages 279 and 280 of *MATHPOWER™ 10* and work through Examples 1 and 2.

4. Do questions 1, 3, 11, 13, 15, and 17 of "Practice" on page 282 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Turn to pages 280 and 281 of *MATHPOWER™ 10* and work through Example 3.

5. Do questions 21, 25, 28, and 38 of "Practice" on page 282 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Linear equations can be used to model many everyday situations. The point-slope form is a useful tool in establishing these equations.

Turn to pages 281 and 282 of *MATHPOWER™ 10* and work through Example 4.

6. Answer questions 41 and 46 of "Applications and Problem Solving" on page 283 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 3.

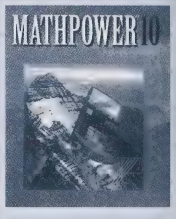
One of the special cases of the point-slope form occurs when the given point lies on the y-axis. The y-coordinate of that point is called the **y-intercept**. You will be using your graphing calculator to investigate this case. Before graphing using your calculator, you may wish to select "5:ZSquare" from the ZOOM menu so that the slopes of your lines will look the same as they would if you did the activity on graph paper.

Turn to "TECHNOLOGY" on page 284 of *MATHPOWER™ 10* and read "Graphing Calculators and Families of Lines."

7. Answer the following questions on pages 284 and 285 of the textbook:

a. questions 1 and 2 of Investigation 1, "Slope and y-Intercept"
b. questions 1 to 5 of Investigation 2, "A Family of Lines"
c. questions 1 to 5 of Investigation 3, "A Different Family of Lines"
d. questions 1 to 4 of Investigation 4, "Related Lines"

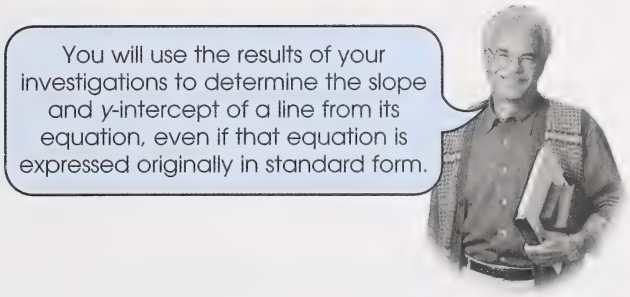
Compare your responses with those in the Appendix, Section 1: Activity 3.

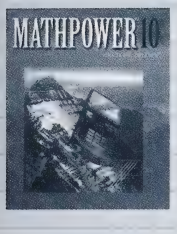


Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

y - intercept: the y - coordinate of the point where a line or curve intersects the y - axis


You will use the results of your investigations to determine the slope and y-intercept of a line from its equation, even if that equation is expressed originally in standard form.



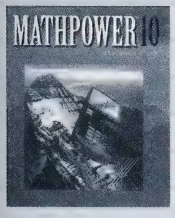
Turn to page 286 of *MATHPOWER™ 10* and read "Linear Equations: Slope and *y*-Intercept Form," including the part titled "Explore: Use the Graphs."

8. Answer the following questions on pages 286 and 287 of the textbook:

- **a.** questions a. to e. of "Explore: Use the Graphs"
- **b.** questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 3.

In your work with linear equations, the two most common forms in which equations are given are standard form $(Ax + By + C = 0)$ and the **slope and *y*-intercept form** $(y = mx + b)$. In the next three examples, you will develop and apply the slope and *y*-intercept form.



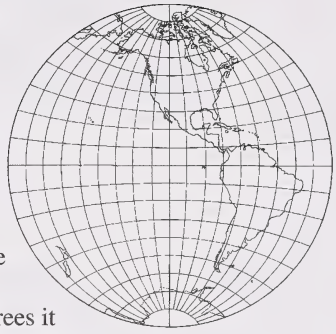
Turn to pages 287 and 288 of *MATHPOWER™ 10* and work through Examples 1 to 3.

9. Answer the following questions on pages 288 and 289 of the textbook:

- **a.** questions 1 and 3 of "Practice"
- **b.** questions 9, 11, and 13 of "Practice"
- **c.** questions 19, 23, and 26 of "Practice"
- **d.** questions 27 and 29 of "Practice"
- **e.** questions 34 and 36 of "Practice"
- **f.** questions 37.a., 38.a., 41, and 44 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 3.

Now Try This



Points on Earth's surface are identified using a spherical coordinate system. Look at a globe on the right.

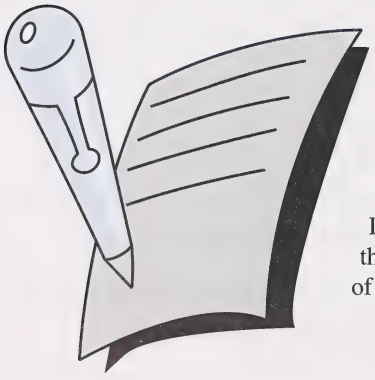
The north-south lines that pass through the poles are called meridians of longitude. Each meridian is identified by the number of degrees it is west or east of the prime meridian, which passes through Greenwich, England. For instance, the position of the Alberta-Saskatchewan border is 110° west longitude. The east-west lines that encircle Earth are called parallels of latitude. Each parallel is identified by the number of degrees it is north or south of the equator. For instance, the position of the longest portion of the border between Canada and the US is 49° north latitude.

10. Find the location and give the meridian of longitude and the parallel of latitude for each of the following:

a. Saskatoon, Saskatchewan
b. Sarnia, Ontario
c. your community

Compare your responses with those in the Appendix, Section 1: Activity 3.

Looking Back



In this activity, you studied procedures for determining the equation of a line, given information that uniquely determines that line. In particular, you applied the point-slope form and the slope and y-intercept form of a linear equation.

In this activity, what kinds of questions did you find the most difficult? In your journal, record an example of each kind together with its solution.

Mathematical Process

- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

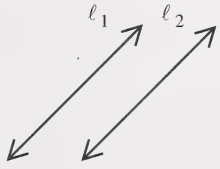
Activity 4: Parallel and Perpendicular

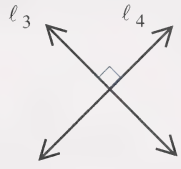
Mathematical Process

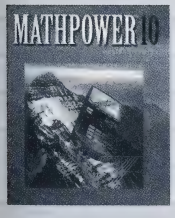
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

The power lines in the photograph trace parallel lines against a spectacular sky. Design and architectural forms that do not contain parallel and perpendicular lines are rare rather than the norm. Take a quick look around your room. Quickly find a dozen objects that contain parallel and perpendicular lines. Are there any objects that don't have such lines?

The symbol for *parallel* is $\|$, and the symbol for *perpendicular* is $\perp$. In the following diagrams, $\ell_1 \| \ell_2$ and $\ell_3 \perp \ell_4$. You will see, in the next part of the activity, how the concepts of parallel and perpendicular are related to slope.







Turn to page 291 of *MATHPOWER™ 10* and read "Parallel and Perpendicular Lines."

1. Answer the following questions on pages 291 and 292 of the textbook:

a. questions a. to c. of "Explore: Use the Graphs—Parallel Lines"
b. questions a. to c. of "Explore: Use the Graphs—Perpendicular Lines"
c. questions 1 to 6 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 4.

Make sure you use the relationship $m_1 \times m_2 = -1$ only for perpendicular, oblique lines. The slope of a horizontal line is 0, and the slope of a vertical line is undefined. It would not make sense to write $0 \times \text{undefined} = -1$.

Also, $m_1 \times m_2 = -1$ may be written as follows:

$$m_1 = \frac{-1}{m_2} \quad \textbf{or} \quad m_2 = \frac{-1}{m_1}$$

When written in this form, you can see that the slopes of perpendicular, oblique lines are negative reciprocals of each other.

Turn to pages 292 and 293 of *MATHPOWER™ 10* and work through Examples 1 and 2.

2. Answer the following questions on pages 294 and 295 of the textbook:

- **a.** questions 1 to 3 of "Practice"
- **b.** questions 11, 14, and 16 of "Practice"
- **c.** questions 28 and 34 of "Practice"
- **d.** questions 40 to 42 of "Practice"
- **e.** questions 44 and 45 of "Practice"
- **f.** questions 53, 59, and 61 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 4.

Now Try This

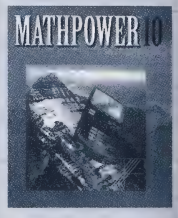
If you have travelled east or west in Canada, you likely will have crossed time zones. A traveller from Peace River to Ottawa loses two hours, but a traveller from Ottawa to Peace River gains two hours. But have you ever wondered what happens when a traveller goes to the North Pole?

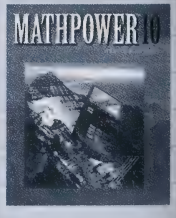
3. Turn to "PROBLEM SOLVING" on page 303 of *MATHPOWER™ 10* and answer question 1 of "Data Bank."

Compare your responses with those in the Appendix, Section 1: Activity 4.

Mathematical Process

- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [ ] Visualization





Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

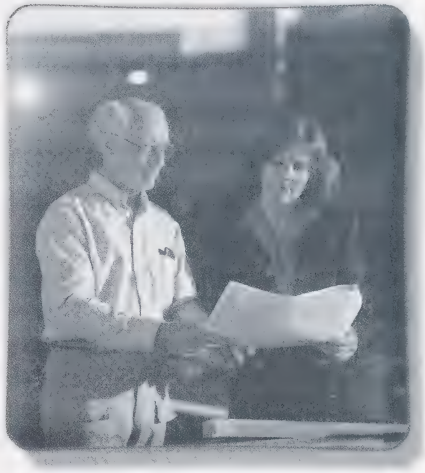
Looking Back

In this activity, you solved problems using slopes of parallel lines and perpendicular lines.

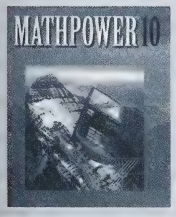
In your journal, record the relationships between the slopes of parallel lines and the slopes of perpendicular lines. What must you consider when working with horizontal and vertical lines?

Activity 5: Graphing Linear Equations

Mathematical Process
- [ ] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization



Urban planners must be able to read maps and blueprints that represent the shape and location of real objects using symbols and scale drawings. Students of mathematics must be able to visualize linear equations and draw their graphs. Graphing calculators will graph equations for you. However, like an urban planner, you must understand the underlying concepts of a graphic representation, regardless of whether that representation appears on paper or on a screen!



Turn to page 296 of *MATHPOWER™ 10* and read "Graphing Linear Equations."

1. Answer the following questions on page 296 of the textbook:

   a. questions a. to d. of "Explore: Draw a Graph"
   b. questions 1 to 6 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 5.

Next, you will review graphing linear equations using the following:

- the ***x*-intercept** and *y*-intercept
- any two points
- the slope and *y*-intercept

x-intercept: the x-coordinate of the point where a line or curve intersects the x-axis

Turn to page 297 of *MATHPOWER™ 10* and work through Examples 1 and 2.

2. Do questions 1, 3, 5, 9, 13, and 14 of "Practice" on page 298 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 5.

So far in this activity, you have graphed oblique lines. You will now investigate horizontal and vertical lines.

Example: Horizontal Lines

Find the equation of the horizontal line through $(1, 3)$; then graph the line.

Solution

Use the point-slope form.

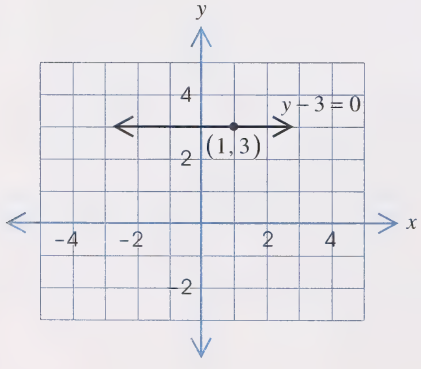
$$y - y_1 = m(x - x_1)$$

$$y - y_1 = 0(x - x_1)$$ ← Since the slope of a horizontal line is 0, $m = 0$.

$$y - y_1 = 0$$

$$y - 3 = 0$$ ← Substitute $y_1 = 3$, since (1, 3) is on the line.

The graph is as follows:



Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization

Notice that the line on the preceding graph has a y-intercept of 3, which is the same as the y-coordinate of the given point. This result will be similar for all horizontal lines. Thus, the form for the equation of a horizontal line is $y-b=0$ or $y=b$.

Now, do the following questions:

3. Find the equation of the horizontal line that passes through each of the following points:

a. $(1,-3)$ **b.** $(0,4)$ **c.** $(0,0)$

4. Graph each of the following horizontal lines, and identify the y-intercept on the graph:

a. $y-2=0$ **b.** $y=-5$ **c.** $3y-7=0$

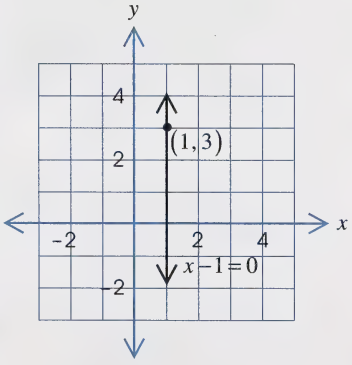
5. What is the equation of the x-axis?

Compare your responses with those in the Appendix, Section 1: Activity 5.

Example: Vertical Lines

Graph the vertical line through $(1,3)$; then find the equation.

Solution



Because the slope of a vertical line is undefined, you can't use either the point-slope form or the slope and y-intercept form. However, notice that all the points the vertical line passes through have the same x-coordinate. For instance, $(1,3)$, $(1,2)$, $(1,1)$, $(1,0)$, $(1,-1)$, and $(1,-2)$ have the same x-coordinate.

Since the x-coordinate $=1$, the equation of the line is $x=1$ or $x-1=0$.

Notice that the line on the preceding graph has an x-intercept of 1, which is the same as the x-coordinate of the given point. This result will be similar for all vertical lines. Thus, the form for the equation of a vertical line is $x - a = 0$ or $x = a$.

Now, do the following questions:

6. Find the equation of the vertical line that passes through each of the following points:

a. $(-1, -3)$ **b.** $(5, 0)$ **c.** $(0, 0)$

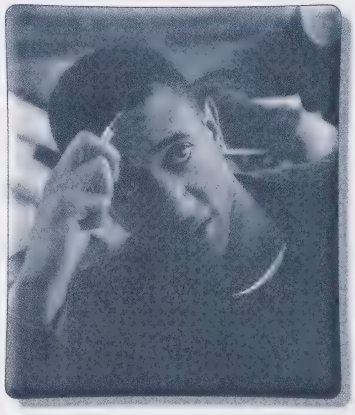
7. Graph each of the following vertical lines, and identify the x-intercept on the graph:

a. $x - 2 = 0$ **b.** $x = -5$ **c.** $3x - 7 = 0$

8. What is the equation of the y-axis?

Compare your responses with those in the Appendix, Section 1: Activity 5.

Students often confuse the equations of the coordinate axes. Remember, for the x-axis, every point has a zero y-coordinate. The point $(3, 0)$, for example, lies on the x-axis. The equation of the x-axis is $y = 0$. For the y-axis, every point has a zero x-coordinate. The point $(0, 3)$, for example, lies on the y-axis. The equation of the y-axis is $x = 0$.



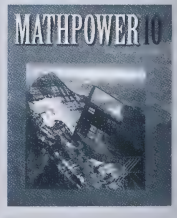
9. Turn to page 298 of *MATHPOWER™ 10* and do the following:

a. questions 26 and 28 of "Practice"
b. questions 29 to 32 of "Applications and Problem Solving"

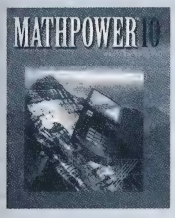
Compare your responses with those in the Appendix, Section 1: Activity 5.

Mathematical Process

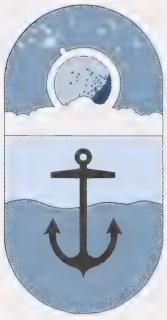
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization



Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Now Try This



Would you like a career that involves going to sea? The oceans have long held a fascination for people from the Prairies. During the Second World War, many of the sailors in the Royal Canadian Navy were from Alberta, Saskatchewan, and Manitoba.

Oceanography is a career that involves the study of the seas and their plant and animal life. Turn to "CAREER CONNECTION" on page 290 of *MATHPOWER™ 10* and read "Oceanography."

10. Do questions 1 to 6 of Investigation 1, "Antarctic Ice" on page 290 of the textbook. **Note:** For question 2.b., the area of Canada is about 10 000 000 km^2.

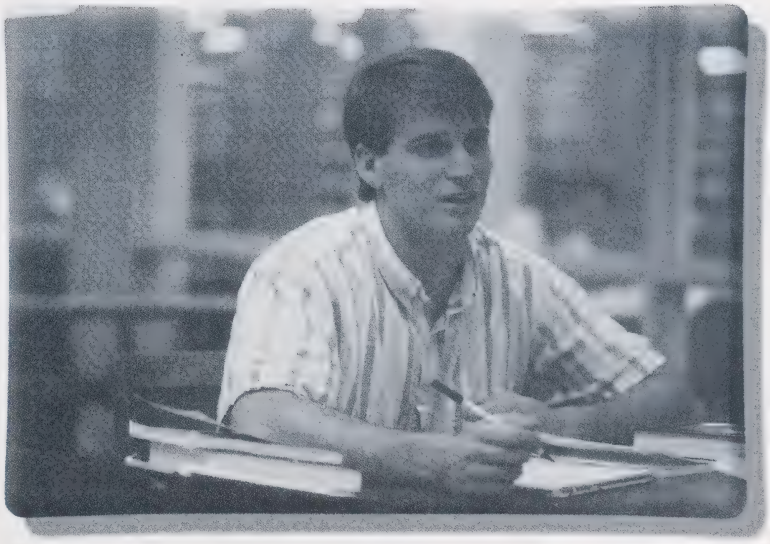
Compare your responses with those in the Appendix, Section 1: Activity 5.

Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Looking Back

In this activity, you graphed a line using any two points, using x- and y-intercepts, and using the slope and y-intercept.

Which method of graphing linear equations do you find the easiest? Can you think of linear equations that you could not graph using that method? Record your thoughts in your journal.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

You have used both the point-slope formula and the slope and *y*-intercept formula to find linear equations. Usually, these formulas are used in the following situations. However, it is possible to use either formula in all three situations.

Formula	Given Information
$y - y_1 = m(x - x_1)$	one point and the slope of the line **or** two points on the line
$y = mx + b$	the slope and *y*-intercept of the line

Example

A line passes through $(1, 2)$ with slope 3. Determine its equation in standard form.

Solution

Method 1: Using the Point-Slope Formula

Let $(x_1, y_1) = (1, 2)$ and $m = 3$.

$$\therefore y - y_1 = m(x - x_1)$$
$$y - 2 = 3(x - 1) \quad \text{Substitute the values for } x_1, y_1, \text{ and } m.$$
$$y - 2 = 3x - 3 \quad \text{Remove the parentheses.}$$
$$3x - 3 = y - 2 \quad \text{Interchange the left and right sides.}$$
$$3x - 3 - y = y - 2 - y \quad \text{Subtract } y \text{ from both sides.}$$
$$3x - y - 3 = -2$$
$$3x - y - 3 + 2 = -2 + 2 \quad \text{Add 2 to both sides.}$$
$$3x - y - 1 = 0$$

Method 2: Using the Slope and *y*-Intercept Formula

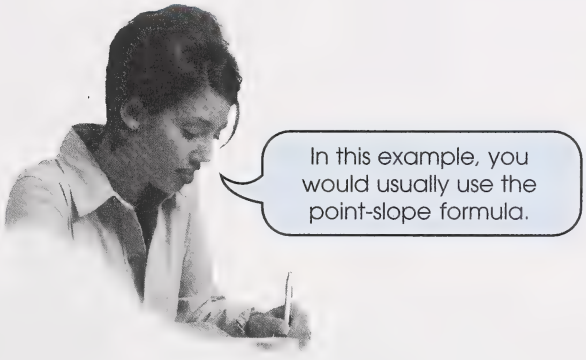
Because $m = 3$, $y = 3x + b$.

Now, because $(1, 2)$ is a point on the line, the coordinates must satisfy the equation. Therefore, substitute $x = 1$ and $y = 2$ into the equation and solve for b.

$$y = 3x + b$$
$$2 = 3(1) + b$$
$$2 = 3 + b$$
$$3 + b = 2 \quad \text{◄— Interchange the left and right sides.}$$
$$3 - 3 + b = 2 - 3 \quad \text{◄— Subtract 3 from both sides.}$$
$$b = -1$$

Substitute $m = 3$ and $b = -1$ into the slope and *y*-intercept formula, $y = mx + b$.

$$y = mx + b$$
$$y = 3x + b$$
$$y = 3x - 1$$
$$3x - 1 = y \quad \text{◄— Interchange the left and right sides.}$$
$$3x - 1 - y = y - y \quad \text{◄— Subtract } y \text{ from both sides.}$$
$$3x - y - 1 = 0$$



Example

Determine the equation, in standard form, of a line with slope 2 and y-intercept 3.

Solution

Method 1: Using the Slope and y-Intercept Formula

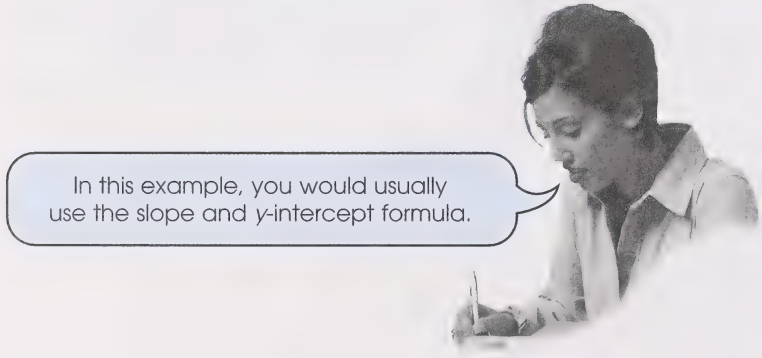
$m = 2$ and $b = 3$

$$\begin{aligned} \therefore\ y &= mx + b \\ y &= 2x + 3 && \text{Substitute.} \\ 2x + 3 &= y && \text{Interchange the sides.} \\ 2x - y + 3 &= 0 && \text{Subtract } y \text{ from both sides.} \end{aligned}$$

Method 2: Using the Point-Slope Formula

Because the y-intercept is 3, the line passes through the y-axis at $(0, 3)$. Let $(x_1, y_1) = (0, 3)$ and $m = 2$.

$$\begin{aligned} \therefore\ y - y_1 &= m(x - x_1) \\ y - 3 &= 2(x - 0) && \text{Substitute the values given.} \\ y - 3 &= 2x && \text{Simplify.} \\ 2x &= y - 3 && \text{Interchange the sides.} \\ 2x - y &= -3 && \text{Subtract } y \text{ from both sides.} \\ 2x - y + 3 &= 0 && \text{Add 3 to both sides.} \end{aligned}$$



Now, try these questions. Do each question two different ways.

1. Find the equation, in standard form, of the line that passes through $(-1, 2)$ and $(1, 1)$.

2. Find the equation, in standard form, of the line with slope -2 and y-intercept 6.

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

Enrichment

Earlier in this section, you found the distance between two points using the distance formula. Did you know that you can program your graphing calculator to find the distance automatically by simply entering the coordinates of the two points?

Turn to "TECHNOLOGY" on page 274 of *MATHPOWER™ 10* and enter the program described in Investigation 1, "Distance Between Two Points."

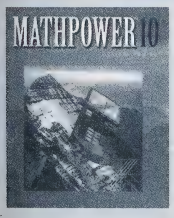
Begin by selecting the program menu by pressing PRGM.

Highlight the NEW menu (as shown in the first display), using the arrow keys to move the cursor, and press ENTER.

At this point, the calculator prompts you to enter the name of the program (as shown in the second display).

Now, name your program. You may call your program "DISTANCE." Once the program has been named, press ENTER.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



You are now ready to enter the program as shown on page 274. To do so, press the following keys.

PRGM [Select the I/O menu.] 8 (8:ClrHome) ENTER

PRGM [Select the I/O menu.] 3 (3:Disp) ALPHA ["] ALPHA [X] 1 2nd [TEST] 1 (1:=) ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 (1:Input) ALPHA [P] ENTER

PRGM [Select the I/O menu.] 3 (3:Disp) ALPHA ["] ALPHA [Y] 1 2nd [TEST] 1 (1:=) ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 (1:Input) ALPHA [Q] ENTER

PRGM [Select the I/O menu.] 3 (3:Disp) ALPHA ["] ALPHA [X] 2 2nd [TEST] 1 (1:=) ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 (1:Input) ALPHA [R] ENTER

PRGM [Select the I/O menu.] 3 (3:Disp) ALPHA ["] ALPHA [Y] 2 2nd [TEST] 1 (1:=) ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 (1:Input) ALPHA [S] ENTER ALPHA [R] − ALPHA [P] STO▸ ALPHA [X] ENTER

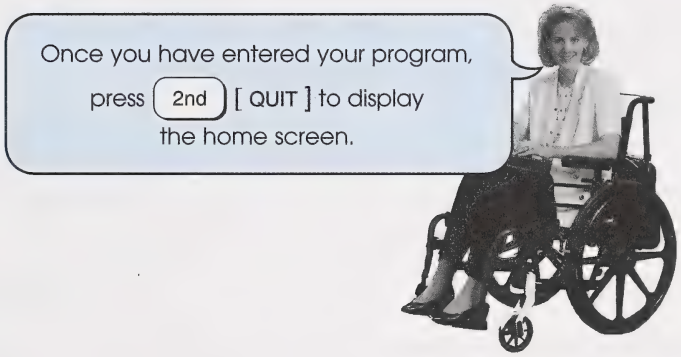
ALPHA [S] − ALPHA [Q] STO▸ ALPHA [Y] ENTER

2nd [$\sqrt{\ }$] ALPHA [X] x^2 + ALPHA [Y] x^2) STO▸ ALPHA [D] ENTER

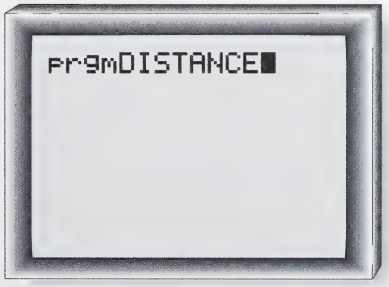
PRGM [Select the I/O menu.] 3 (3:Disp) 2nd ALPHA ["] [D] [I] [S] [T] [A] [N] [C] [E] ["] ENTER

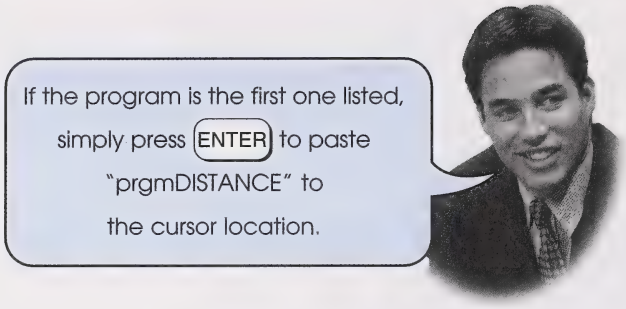
PRGM [Select the I/O menu.] 3 (3:Disp) 2nd ALPHA ["] [I] [S] ["] ENTER

PRGM [Select the I/O menu.] 3 (3:Disp) ALPHA [D] ENTER

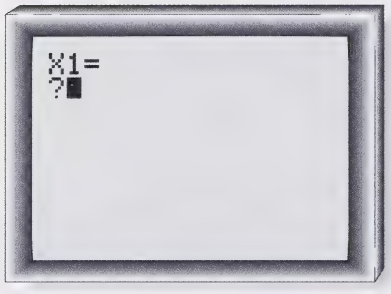


To run your program, press PRGM and select "DISTANCE" (or the name you used) from the EXEC menu. Your display should look as follows:





Press ENTER a second time in order to activate the program. Your display should look as follows:



At this point, the program prompts you to enter the x-coordinate of the first point. Enter that value and press ENTER. Continue to enter the values of the remaining coordinates the program prompts you to supply—the y-coordinate of the first point and the x- and y-coordinates of the second point.

Turn to "TECHNOLOGY" on page 274 of *MATHPOWER™ 10* and answer the following:

1. questions 1 and 2 of Investigation 1, "Distance Between Two Points"

2. questions 1 and 2 of Investigation 2, "Midpoint of a Line Segment"

Note: To program your calculator to find the midpoint of a segment, all you will have to do is change a few lines of the distance program.

Compare your answers with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

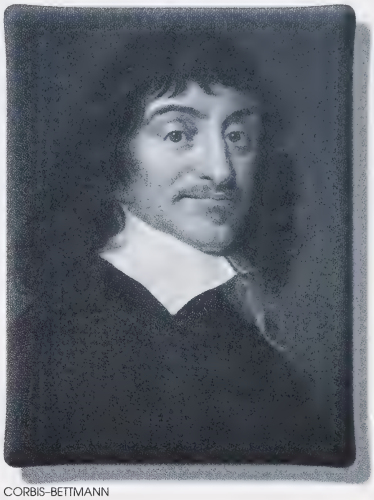
Conclusion

In this section, you solved problems involving the distances between points in the coordinate plane, the midpoints of segments, and the slopes of lines. You studied the relationships between parallel lines and the relationship between perpendicular lines. You then graphed the equations of lines on grid paper and on a graphing calculator and determined how to find linear equations from information that uniquely defines the lines. In each instance, you investigated applications to the real world.

A French mathematician, named René Descartes (1596–1650), is credited with devising the coordinate plane and coming up with the idea of graphing equations using sets of points. In recognition, the coordinate plane is often called the Cartesian plane.

If you have access to the Internet, go to the following website and find out more about René Descartes:

http://www-groups.dcs.st-and.ac.uk ~history/Mathematicians/Descartes.html



CORBIS–BETTMANN

René Descartes

Assignment

Turn to Assignment Booklet 2A and complete the assignment for Section 1.

Section 2

Relations and Functions

Of the family members in the photograph, pick out any two individuals. Can you describe the relationship between them? Did you choose the father and the daughter? the mother and the son? the brother and sister? The human mind always searches for relationships in order to help it interpret the external world.

Mathematics includes the search for patterns and relationships. You are already familiar with the use of graphs and formulas as tools for modelling those relationships.

In this section, you will investigate mathematical relationships, called relations. You will discover that some of these relations can be classified as functions. Further, you will study the properties of relations and functions and determine how to represent them graphically. You will also discover that an important subset of these relations and functions have linear graphs and that they can be used to describe many natural phenomena. Other relations will have non-linear graphs. They, too, have an important role in describing the world around you.

Activity 1: Relations

How does the partially filled bottle of milk on the right compare with the one on the left? Is the relationship between the amounts of milk the same as that between the number of eggs?

Mathematicians call a comparison between two sets of elements a **binary relation** or simply a **relation**. A relation can be represented using ordered pairs.

relation: a set of ordered pairs

Turn to page 212 of *MATHPOWER™ 10* and read "Binary Relations."

1. Answer the following questions on page 212 of the textbook:

a. "Explore: Look for a Pattern"
b. questions 1 to 5 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 1.

In the questions you just completed, the relation described the correspondence between the length of the side of the large square and the number of red squares. That relation consisted of the ordered pairs $(3,8)$, $(4,12)$, $(5,16)$, $(6,20)$, $(7,24)$, ...

domain: the set of first elements of all ordered pairs of a relation

The set of first elements of the ordered pairs is $\{3,4,5,6,7, \ldots\}$ and refers to the lengths of the sides of the large squares. The set of first elements selected from the ordered pairs of a relation is called the **domain** of the relation.

range: the set of second elements of all ordered pairs of a relation

The set of second elements of the ordered pairs is $\{8,12,16,20,24,\ldots\}$ and refers to the number of red squares. The set of second elements selected from the ordered pairs of a relation is called the **range** of the relation.

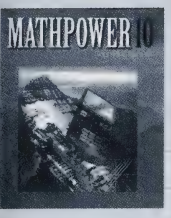
Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

An easy way of remembering that domain refers to first elements and that range refers to second elements is that **d** for **d**omain comes **before r** for **r**ange in the alphabet.

A nineteenth-century mathematician, named Johann Peter Gustav Lejeune Dirichlet (1805–1859), introduced the terms domain and range into the study of relations. If you have access to the Internet, you may wish to find out more about Johann Dirichlet at the following website:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Dirichlet.html



Turn to page 212 of *MATHPOWER™ 10* and read from the red line to the red line on page 213. Work through "Example: The Gemini Project" on page 213.

2. Answer the following questions on pages 214 and 215 of the textbook:

a. questions 1 and 3 of "Practice"
b. questions 5 and 7 of "Practice"
c. questions 9, 11, and 13 of "Practice"
d. questions 17 and 18 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process
- Communication
- Connection
- Estimation
- [x] Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Now Try This

When working with relations, the ability to mentally evaluate algebraic expressions that form part of an equation is a time-saver; it also gives you insight as to whether a particular answer is reasonable or not.

3. Turn to page 209 of *MATHPOWER™ 10* and answer questions 1 to 10 of "Mental Math: Expressions and Equations."

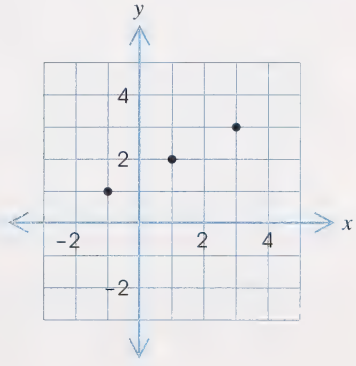
Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Looking Back

In this activity, you identified a relation and described that relation in terms of ordered pairs (a rule in word or equation form or a graph). You also determined the domain and range of a relation from its ordered pairs or from its graph.

The graph on the right is a graph of a relation. In your journal, describe how would you communicate this graph to a friend over the phone.

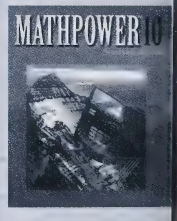


Activity 2: Linear Relations

When you travel long distances by car, do you ever count the hours until you arrive at your destination? You can calculate the average speed for the trip by dividing the distance travelled by your traveling time.

Is the relationship between distance and time a linear relation? What would the shape of the distance-time graph be if your speed varied? if your speed were constant? In this activity, you will be exploring these and other questions about relations!

Turn to page 216 of *MATHPOWER™ 10* and read "Linear Relations and the Line of Best Fit."



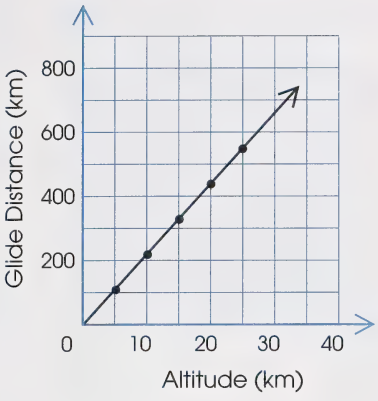
1. Answer the following questions on page 216 of the textbook:

a. "Explore: Draw a Graph"
b. questions 1 and 2 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 2.

The relation graphed in your exploration is an example of a **linear relation** because the points of its graph lie along a straight line. The slope is constant (22 km for every km of altitude).

linear relation: a relation between two values whose graph is along a straight line



Be careful not to extend the graph any farther than the origin on the left since neither the glide distance nor the altitude can be negative. In this situation, the domain may be expressed as altitude ≥ 0 km and the range as glide distance ≥ 0 km.

Because the graph is continuous in the first quadrant, you can use it to **interpolate**. That is, you can find values between the ordered pairs that were originally used in the table from which the graph was drawn. You can also **extrapolate**. That is, find values outside the given ordered pairs in the table.

interpolate: to use the graph of a relation to predict values between the given ordered pairs

extrapolate: to use the graph of a relation to predict values beyond the given ordered pairs

Example

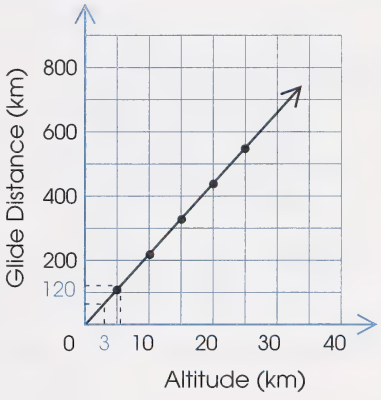
Using the graph of gliding distance versus altitude, determine the following:

a. the distance the aircraft would glide from an altitude of 3 km
b. the altitude from which the plane could glide 120 km

Solution

a. This is an example of extrapolation because the lowest altitude given in the table was 5 km.

The answer is obtained by going straight up to the graph from 3 km on the horizontal axis, then straight across to the vertical axis. The gliding distance is read from the vertical axis. It appears to be approximately 70 km. The actual value is 66 km, since the plane glides 22 km for every 1 km of altitude.

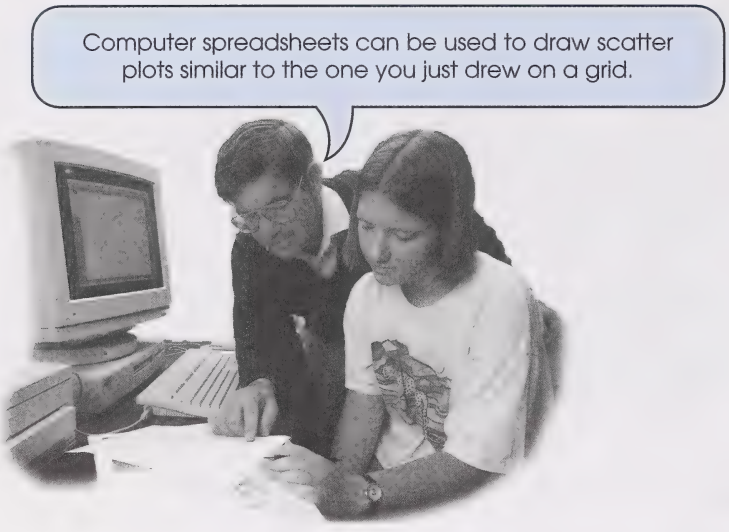


b. This is an example of interpolation because 120 km lies between 110 km and 220 km, both of which were in the table.

The answer is obtained by going straight across to the graph from 120 km on the vertical axis, then straight down to the horizontal axis. The altitude is read from the horizontal axis. It appears to be approximately 6 km. The actual value is $120 \div 22 \doteq 5.45$ km, since the plane glides 22 km for every 1 km of altitude.

2. Turn to page 217 of *MATHPOWER™ 10* and answer "Explore: Draw a Graph."

Compare your response with the one in the Appendix, Section 2: Activity 2.



If you are using *ClarisWorks™ 4.0*, follow these steps:

Step 1: Open a new spreadsheet. Cell A1 will be highlighted. Type the heading "Building" (as shown in the table in question 2); then press "return." In cell A2, type "First Canadian Place, Toronto"; then press "return." Continue until every building name has been entered.

Note: You will probably need to adjust the column and row widths.

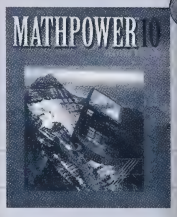
Step 2: Use your mouse to activate cell B1. Enter the heading and values (as shown in the table).

Step 3: Starting with cell C1, enter the third column (as shown in the table).

Step 4: Highlight columns B and C by using the mouse. Position the cross on B; click and hold and drag the cross to C; then release the mouse.

Step 5: From the Options Menu, select "Make Chart."

Step 6: Lastly, select X-Y Scatter; then click on "OK."

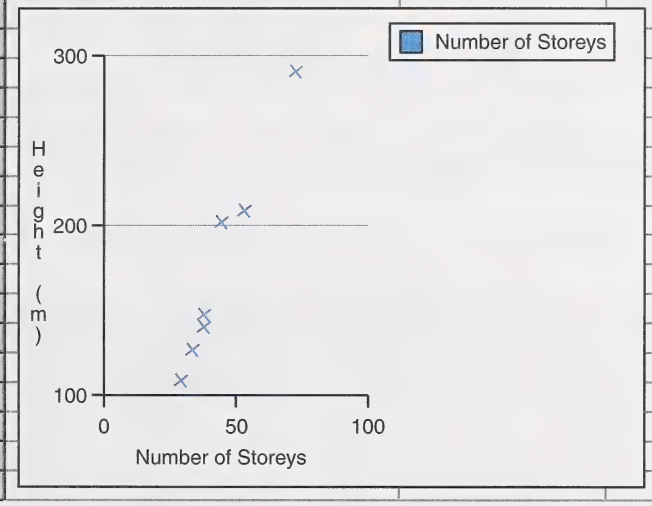


Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

The table and graph should look somewhat as follows:

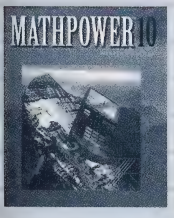
	A	B	C
1	**Building**	**Number of Storeys**	**Height (m)**
2	First Canadian Place, Toronto	72	290
3	Manulife Place, Edmonton	36	146
4	Petro-Canada Centre, W Tower, Calgary	52	210
5	Place de Ville, Tower C, Ottawa	29	112
6	Royal Centre Tower, Vancouver	36	140
7	1100 Rue de la Gauchetière, Montreal	45	204
8	Toronto Dominion Centre, Winnipeg	33	126



Now, you can draw the line of best fit if you do not have that feature on your spreadsheet program.

3. Turn to "TECHNOLOGY" on pages 222 and 223 of *MATHPOWER™ 10* and answer questions 1, 2.a., and 3 of Investigation 1, "Members of Parliament."

Compare your responses with those in the Appendix, Section 2: Activity 2.



You have just graphed a scatter plot using a grid and using a computer spreadsheet. Next, you will explore graphing a linear relation using a grid and using a graphing calculator. You will also graph the line of best fit.

Turn to pages 217 and 218 of *MATHPOWER™ 10* and work through Examples 1 and 2.

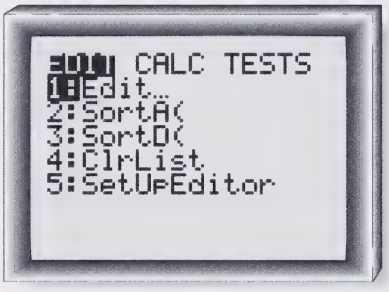
4. Answer the following questions on page 219 of the textbook:

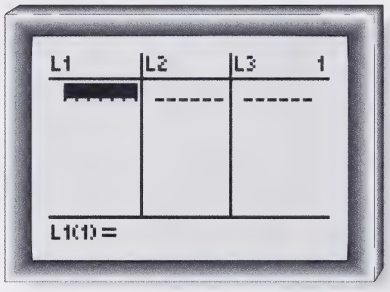
a. questions 1 and 3 of "Practice"
b. questions 12 and 19 of "Practice"
c. question 24 of "Practice"

Compare your responses with those in the Appendix, Section 2: Activity 2.

To graph the ordered pairs from Example 1 on page 217 of *MATHPOWER™ 10* and the line of best fit using your graphing calculator, use the following procedure:

Step 1: Enter the table of values. Press [STAT] and select "1:Edit" from the EDIT menu.





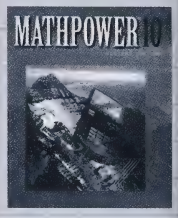
Step 2: Enter the x-coordinates in column L1 as follows:

[3] [ENTER] [2] [ENTER] [1] [ENTER] [0] [ENTER] [(–)] [1] [ENTER]

Step 3: Enter the y-coordinates in column L2 as follows:

Using the arrow keys, move the cursor to column L2 and enter the y-values.

[3] [ENTER] [1] [ENTER] [(–)] [1] [ENTER]
[(–)] [3] [ENTER] [(–)] [5] [ENTER]

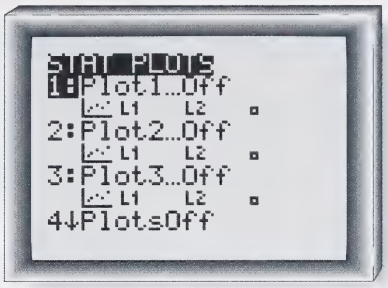


Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [X] Technology
- Visualization

Step 4: Graph the ordered pairs.

First, press 2nd [STAT PLOT]. Make sure the new display looks as follows:



Press 1, which selects "1:Plot 1...Off." Use the arrow keys to move the cursor and press ENTER in order to select the desired options. Either • or □ may be selected to display the point on the screen.

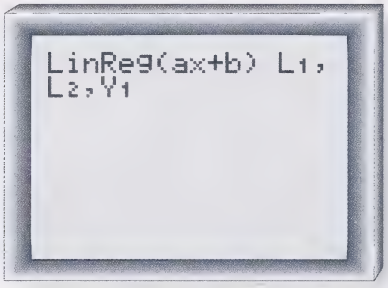
Step 5: Press ZOOM 9 to select "9:ZoomStat." This displays the scatter plot as follows:

In order to draw the line of best fit, you will use the linear regression function. To draw the line of best fit, do the following:

Step 1: Establish the slope and *y*-intercept form of the line of best fit by inputting the following:

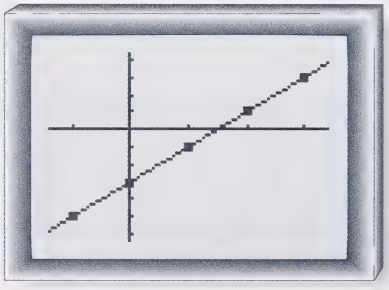
STAT [Select the CALC menu.] 4 (4:LinReg (ax + b)) 2nd 1 (pastes "L1" on the screen) , 2nd 2 (pastes "L2" on the screen) , VARS [Select the Y-VARS menu.] 1 (1:Function...) 1 (1:Y_1) (pastes Y_1 on the screen)

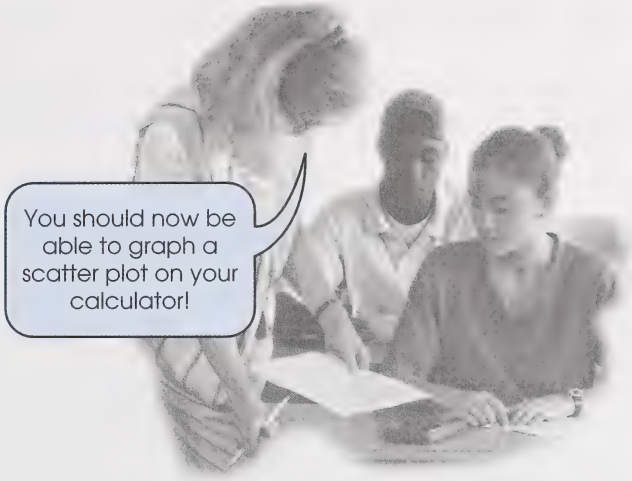
You will now see the following:



Step 2: Press ENTER to display the values of *a* and *b*, the slope and *y*-intercept of the line of best fit.

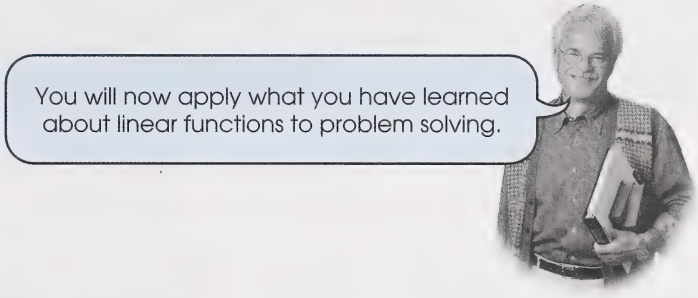
Step 3: Press GRAPH to display the line of best fit.





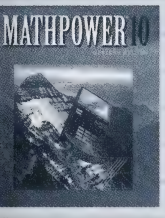
5. Turn to "TECHNOLOGY" on pages 222 and 223 of *MATHPOWER™ 10* and answer question 5.a. of Investigation 1, "Members of Parliament."

Compare your response with the one in the Appendix, Section 2: Activity 2.



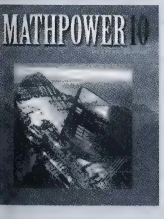
6. Turn to pages 219 and 220 of *MATHPOWER™ 10* and answer questions 26, 28, and 31 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 2: Activity 2.



Mathematical Process

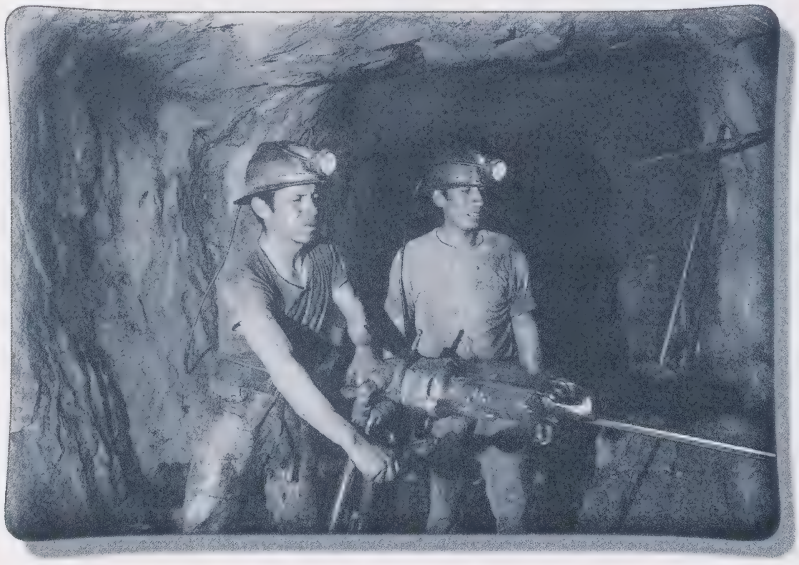
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization



Now Try This

One of the problems of a modern affluent society is trying to balance the need for resources and protection of the environment. Before large forestry, petroleum, or mining projects are allowed to proceed, governments review the results of environmental impact studies. Are you interested in a career that is related to the environment?

Turn to "CAREER CONNECTION" on page 221 of *MATHPOWER™ 10* and read "The Environment."



7. Answer question 1 of Investigation 1, "Gold Mining" on page 221 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

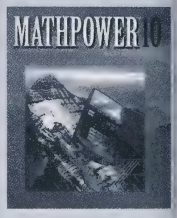
Looking Back

In this activity, you identified linear relations and plotted linear data using graph paper, a graphing calculator, and a spreadsheet software. You also drew lines of best fit from scatter plots. You should now be able to interpret graphs and make predictions from them using the techniques of interpolation and extrapolation.

When graphing scatter plots, what are the advantages and disadvantages of using a graphing calculator and computer spreadsheet software? Record your thoughts in your journal.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization



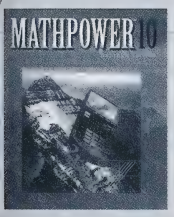
Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 3: Non-Linear and General Relations

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Computer graphics can fascinate and stimulate the imagination. They are central to the creation of special effects in the movie industry. The computer graphics shown in the photograph arise from non-linear relations.



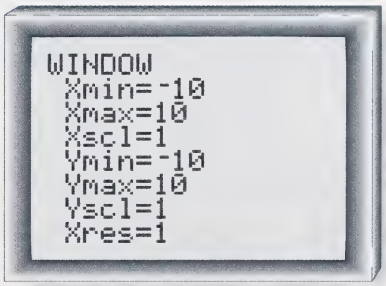
Turn to page 224 of *MATHPOWER™ 10* and read "Non-Linear Relations" and "Explore: Study the Graph."

1. Answer questions 1 to 6 of "Inquire" on page 224 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 3.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

You can graph non-linear relations using your graphing calculator. To begin, you must set the viewing window to suitable values. For the questions in this activity, the standard window will be suitable. Press ZOOM 6. Now press WINDOW. The screen on the right should appear.



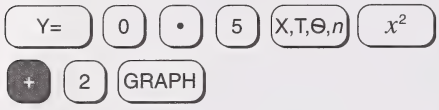
If you have to change this viewing window, ensure that you press (–) for a negative value and that you press ENTER after making the necessary change.

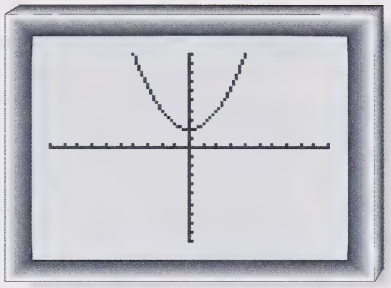
Example

Graph $y = 0.5x^2 + 2$ using a graphing calculator.

Solution

Press the following keystrokes:





Turn to pages 224 and 225 of *MATHPOWER™ 10* and work through Examples 1 and 2.

2. Answer the following questions on pages 225 to 227 of the textbook:

a. questions 1 and 3 of "Practice"
b. questions 5, 6, 9, 10, and 13 of "Applications and Problem Solving"

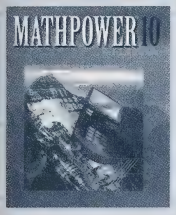
Compare your responses with those in the Appendix, Section 2: Activity 3.

Turn to page 228 of *MATHPOWER™ 10* and read "General Relations" and "Explore: Study the Graph."

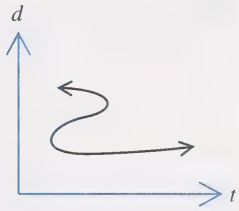
3. Answer questions 1 to 5 of "Inquire" on page 228 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 3.

Mathematical Process
- [x] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [x] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



When interpreting a graph, pay particular attention to the labels on the axes. In a distance-time graph, for instance, where the horizontal axis represents time, the graph will be drawn from left to right. The following graph is impossible because time does not stand still or go backwards!



Turn to pages 228 and 229 of *MATHPOWER™ 10* and work through "Example: Height of Water in a Spa."

4. Answer questions 1, 2, 3, 5, 11, and 12 of "Applications and Problem Solving" on pages 229 to 231 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 3.

Now Try This

Looking for patterns and determining the equations of relations can be made much easier by systematically investigating cases and recording your results in a table.

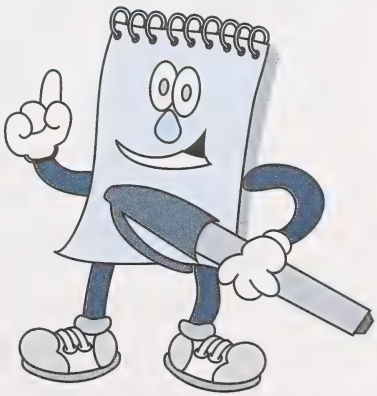
5. Turn to "INVESTIGATING MATH" on page 210 of *MATHPOWER™ 10* and answer questions 1 to 5 of Investigation 1, "Polygons From 8 Squares."

Compare your responses with those in the Appendix, Section 2: Activity 3.

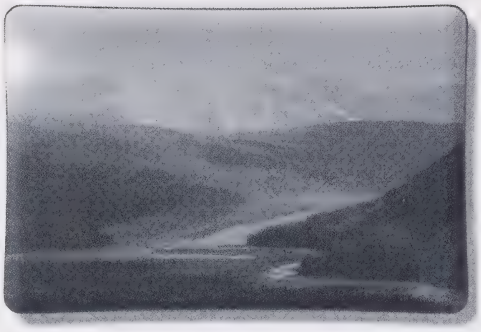
Looking Back

In this activity you graphed non-linear relations and interpreted the graphs of relations that represent real-world situations.

How long did it take you to complete this activity? In your journal, sketch a graph of how long it took you to complete this activity. Describe your graph in words.



Activity 4: Functions



Many scientists around the world are concerned that ice fields and glaciers are retreating more rapidly now than in the past. It is just a matter of time before many disappear altogether. Is the accelerated retreat of the ice a "function" of global warming?

If you have access to the Internet, you can obtain information on the status of the Columbia Icefields and nearby glaciers. The following site describes recent research on how these glaciers have been changing:

http://www1.tor.ec.gc.ca/crysys/index.html

In everyday speech, people use the word "function" to mean that one event depends on the occurrence of another (that the events are related). The income taxes you pay, for instance, are a function of your earnings. But what is a **function** in mathematics? It is a concept that is fundamental to your study of mathematics. First defined by a mathematician named Euler in 1749, a function is a relationship between two variables, where the value of one variable depends on the value of the other variable.

To see a more modern definition, turn to page 232 of *MATHPOWER™ 10* and read "Functions."

1. Answer the following questions on page 232 of the textbook:

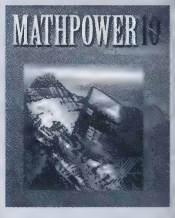
a. "Explore: Complete the Table"
b. questions 1 to 3 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 4.

You have seen that a function is a special kind of relation. Recall that a relation is a set of ordered pairs. No two **distinct** ordered pairs of a function have the same x-coordinate. If $(3, a)$ and $(3, b)$ were ordered pairs of the function, f, then $a = b$; the ordered pairs would not be different—they would represent the same point! If $a \neq b$, then f would not be a function, but simply a relation.

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

function: a relation where no two distinct ordered pairs have the same first element

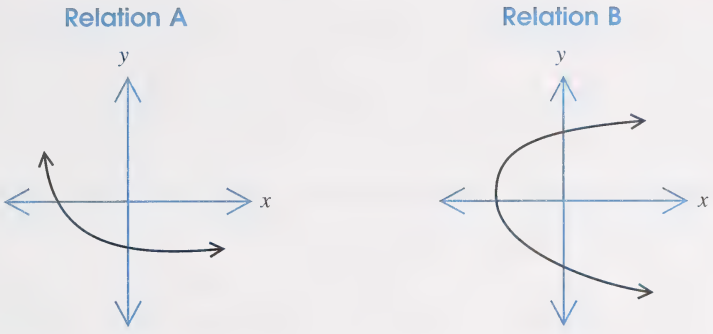


Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization

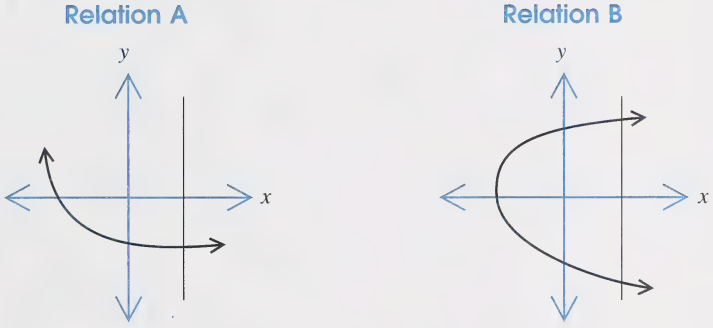
One way to determine whether a relation is a function is to look at its graph.

Example

Which relation is a function? Explain.



Solution



Relation A is a function. No two ordered pairs have the same x-coordinate. If two ordered pairs had the same x-coordinate, they would lie along a vertical line. The graph of a function will not have a vertical line that passes through more than one point. Therefore, Relation A passes the **vertical line test** for a function.

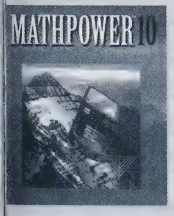
Relation B is not a function. Notice that the vertical line drawn intersects the graph at two points. Therefore, Relation B fails the vertical line test! The two ordered pairs defined by the points of intersection have the same x-coordinate.

2. Turn to page 234 of *MATHPOWER™ 10* and answer questions 1, 3, and 5 of "Practice."

Compare your responses with those in the Appendix, Section 2: Activity 4.

---

vertical line test: a test for determining whether a given graph represents a function



If a relation is a function, a special notational device, called **function notation**, may be used to represent ordered pairs.

function notation: a method of writing a function, f, namely $y = f(x)$

Turn to page 232 of *MATHPOWER™ 10* and read from the red line to the end of Example 2 on page 233, working through Examples 1 and 2.

3. Do questions 7.a. to 7.c., 8.a. to 8.c., 9.a. to 9.c., and 10.a. to 10.c. of "Practice" on page 234.

Compare your responses with those in the Appendix, Section 2: Activity 4.

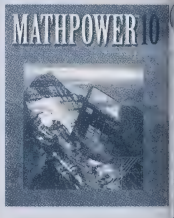
Remember, function notation is a briefer way of expressing statements, such as "When x is 2, the value of function f equals 7." This statement becomes $f(2) = 7$ and means $(2, 7)$ is a point on the graph of function f. You will now investigate how points may be shown using an arrow diagram.

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

Turn to page 233 of *MATHPOWER™ 10* and work through Example 3.

4. Do questions 11, 12, and 14 of "Practice" on page 234 of the textbook.

5. Answer questions 17, 20, 22 to 26, and 31 of "Applications and Problem Solving" on pages 234 and 235 of the textbook.



Compare your responses with those in the Appendix, Section 2: Activity 4.

Now Try This

When evaluating functions using function notation, mental calculations will reduce the number of steps you will have to write down.

Mathematical Process
- Communication
- Connection
- Estimation
- [x] Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Turn to page 209 of *MATHPOWER™ 10* and read "Mental Math: Subtracting Using Compatible Numbers."

6. Answer questions 1 to 12 on page 209 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 4.

Looking Back

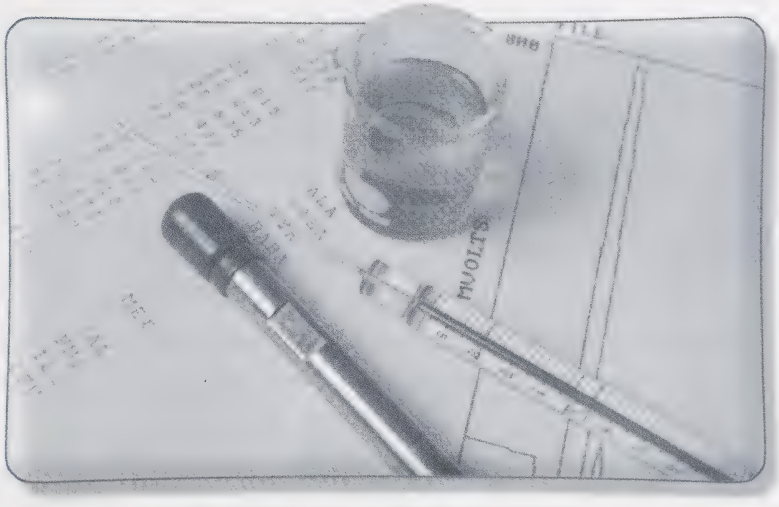
Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

In this activity, you determined whether a relation is also a function, used function notation to evaluate and represent functions, and solved problems that involved situations that could be modelled using functions.

In your journal, describe an everyday situation that can be modelled using a function. Represent that function the following ways:

- a rule in words
- ordered pairs
- a graph
- an equation using function notation

Activity 5: Linear Functions

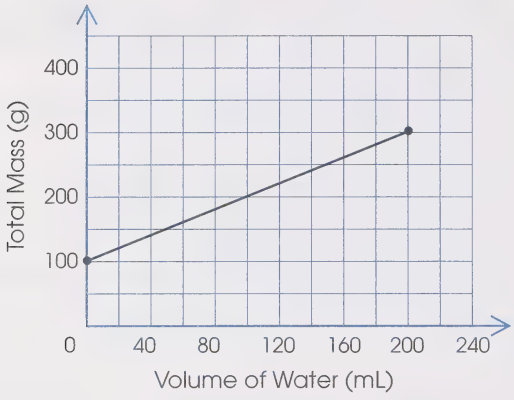


Mathematical Process
- [ ] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Have you considered a career working in a laboratory? Laboratory work involves careful measurement, attention to detail, careful observations, analysis, and accurate records.

Suppose you have a glass beaker, similar to the one in the photograph, that has a mass of 100 g when empty. If the beaker can hold 200 mL of liquid, what will the combined mass of the beaker and its contents be as you gradually fill it with water?

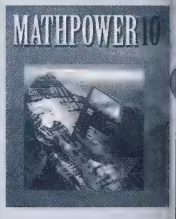
The total mass may be represented as a function of the volume of water in the beaker. Examine the following graph:



Notice that the graph is linear. This function is an example of a **linear function**.

linear function: a function of the form $f(x) = mx + b$

Turn to page 236 of *MATHPOWER™ 10* and read "Applications of Linear Functions"



1. Answer the following questions on page 236 of the textbook:

- **a.** questions a. and b. of "Explore: Draw a Graph"
- **b.** questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 5.

One type of linear function is a **direct variation**. In a direct variation, multiplying one variable changes the other variable by the same factor.

direct variation: a linear function of the form $y = kx$

Turn to page 236 of *MATHPOWER™ 10* and read from the red line to the bottom of page 237, working through Example 1.

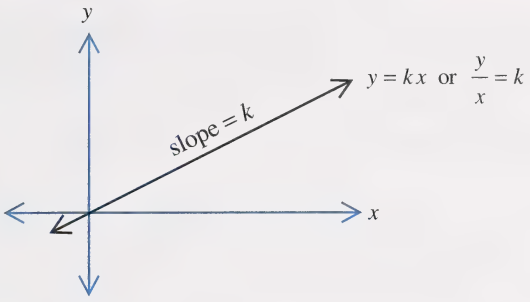
2. Answer questions 1, 3, 5, 7, 9, and 11 of "Practice" on pages 238 and 239.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

Compare your responses with those in the Appendix, Section 2: Activity 5.

The principal features of a direct variation are as follows:



- The ratio of the y-coordinate to the x-coordinate of each ordered pair is constant.

$$\frac{y}{x} = k \text{ or } y = kx$$

- The ratio, k (or constant of variation), is the slope of the line.
- The graph lies along a line that passes through the origin.

The statement "y varies directly as x" may be written using symbols as $y \propto x$.

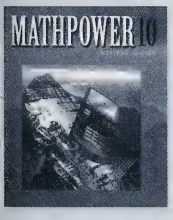
partial variation: a linear function in the form of $y = mx + b$, where $b \neq 0$

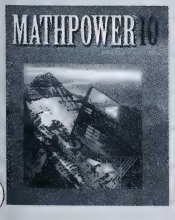
A variation closely related to a direct variation is called a **partial variation**. A partial variation is also a type of linear function.

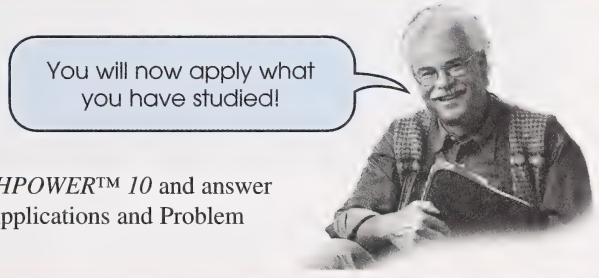
Turn to page 238 of *MATHPOWER™ 10* and read from the top of the page to the red line, working through Example 2.

3. Do questions 13 and 14 of "Practice" on page 239 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 5.







4. Turn to page 239 of *MATHPOWER™ 10* and answer questions 17 and 18 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 2: Activity 5.

Now Try This

As you have seen, working with the graphs of relations and functions helps to visualize and analyse the relationships between two variables. Graphs and diagrams are essential in problem solving and in understanding underlying principles in mathematics.

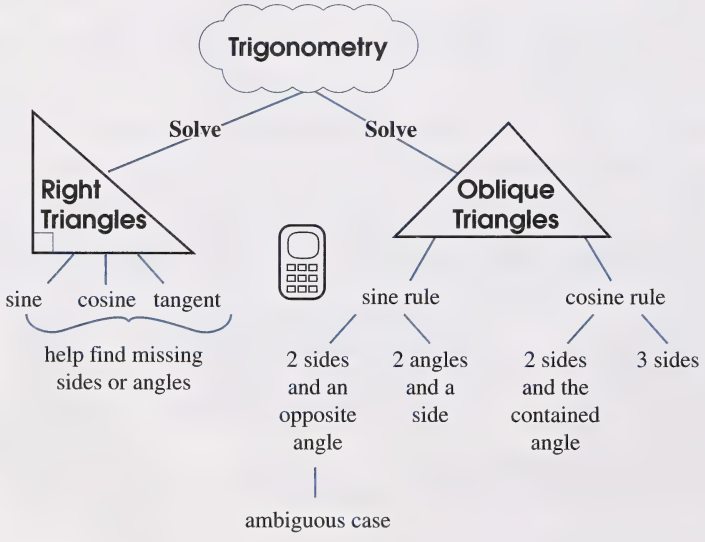
5. Turn to page 240 of *MATHPOWER™ 10* and answer questions a. and b. of "Logic Power."

Compare your answers with those in the Appendix, Section 2: Activity 5.

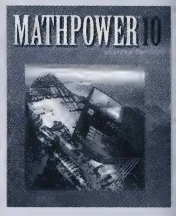
Looking Back

In this activity, you identified linear functions and their graphs. In particular, you worked with two types of linear functions: direct variations and partial variations.

In your journal, draw a "mind map" of what you have learned in this activity. A mind map is a diagram with lines and arrows showing the connections among ideas and processes. It is a blueprint of the concepts you have mastered. As an example, look at a mind map of how a particular student "sees" trigonometry.



Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- [x] Visualization



Mathematical Process
- [x] Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

One of the difficulties you may have when graphing relations and functions using a table of values is deciding whether the points (the ordered pairs) should be joined to form a continuous curve or left as separate points.

Consider the following example of a linear function.

Example

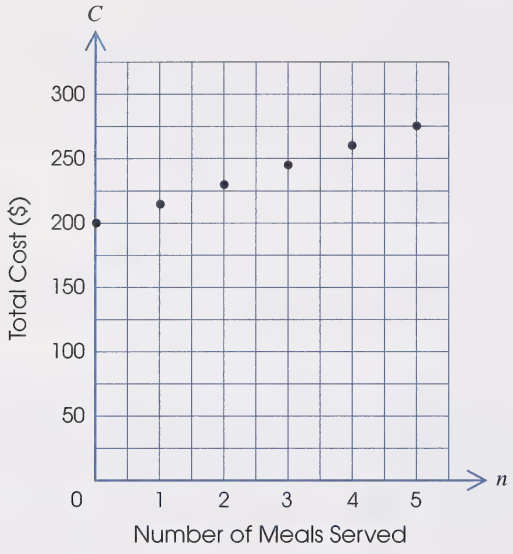
A hotel that caters weddings charges $200 for the use of the dining room, which can hold 300 people, and $15 for each meal served at the reception.

Graph the total cost as a function of the number of meals served.

Solution

Let n be the total number of meals served and C be the total cost.

n	0	1	2	3	4	5
C	200	215	230	245	260	275



$\therefore\ C = 200 + 15n$, where $n = 0, 1, 2, \ldots, 300$

Since there is no charge for partial servings, n must be a whole number. Therefore, the points should not be joined; that is, the domain of this function is $\{0, 1, 2, 3, \ldots, 300\}$.

For questions 1 and 2, decide whether the points in the relation can be joined or not. Justify your answers.

1. The graph is d versus t, where d represents the distance you can cycle in t hours at 10 km/h.

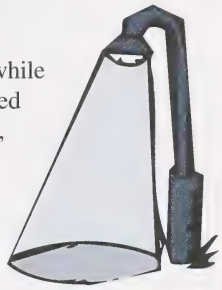
2. The graph is C versus n, where C represents the total cost of purchasing n soft drinks at $1.00/soft drink.

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.

Enrichment

Often, a function or relation consists of a number of sections linked together, each defined by its own equation.

Example



Gretta is standing 2 m from the lamppost at the corner of her street while waiting for her friend to meet her. Bored, she walks for 10 s at a speed of 1 m/s directly away from the post. She stops for 5 s, turns around, and walks towards the post for 5 s at a speed of 0.5 m/s.

Graph her distance, d, (in metres) from the lamppost versus time, t, (in seconds) over this 20-s interval using the following:

a. a grid
b. a graphing calculator

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

Solution

a. The first part of the function is defined as follows:

$$d = 2 + 1t, \text{ when } 0 \le t \le 10$$

After 10 s, Gretta is $2 + 1(10) = 12$ m from the post.

For the next 5 s, because she is standing still, the function is defined as follows:

$$d = 12, \text{ when } 10 < t \le 15$$

For the last 5 s, because she is walking towards the post, the function is defined as follows:

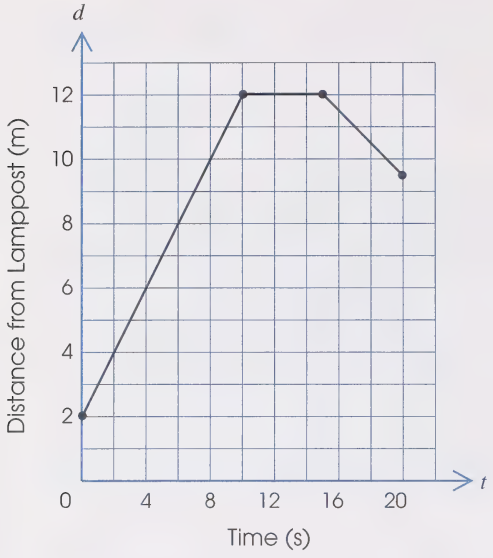
$$d = 12 - 0.5(t - 15), \text{ when } 15 < t \le 20$$

When $t = 20$,

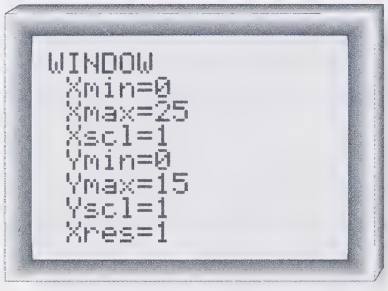
$$\begin{aligned} d &= 12 - 0.5(20 - 15) \\ &= 12 - 0.5(5) \\ &= 12 - 2.5 \\ &= 9.5 \end{aligned}$$

After 20 s, Gretta is 9.5 m from the lamppost.

Piecing these parts together, you obtain the following graph:



b. To graph $d = 2 + 1t$, when $0 \le t \le 10$, first set the viewing window so that the entire graph will appear on the display. For instance, set the values as follows:



Also, the graph type must be set to "Dot" under the MODE menu. At the end of this example, set the MODE menu to "Connected" to see why that graph type is inappropriate. You will notice that the ends of each piece of the graph become joined to points on the horizontal axis!

Now, to enter the first piece of the function, press the following:

Y= (2 + 1 X,T,Θ,n) (0 2nd [TEST]
6 X,T,Θ,n) (X,T,Θ,n 2nd [TEST] 6 1 0)
ENTER

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

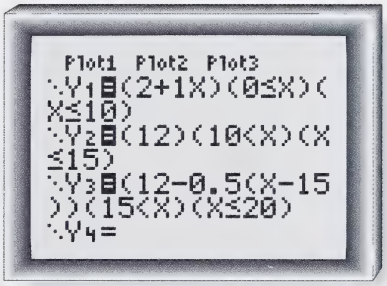
To enter the second piece, press the following:

(1 2) (1 0 2nd [TEST] 5 X,T,Θ,*n*)
(X,T,Θ,*n* 2nd [TEST] 6 1 5) ENTER

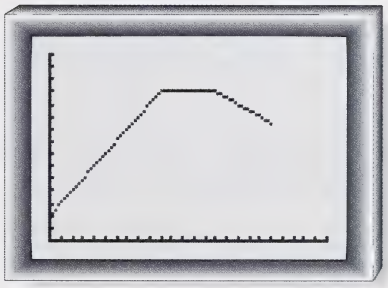
To enter the third piece, press the following:

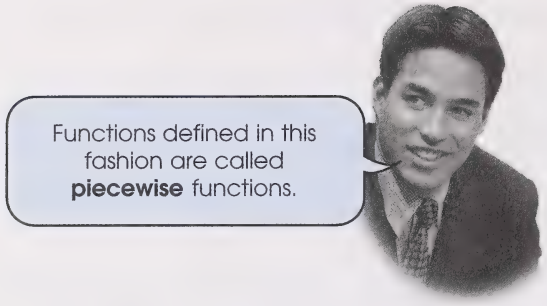
(1 2 (−) 0 • 5 (X,T,Θ,*n* (−) 1 5)
) (1 5 2nd [TEST] 5 X,T,Θ,*n*) (X,T,Θ,*n*
2nd [TEST] 6 2 0) ENTER

After entering the three pieces, your display should look as follows:



Now, press GRAPH. Your display should look as follows:



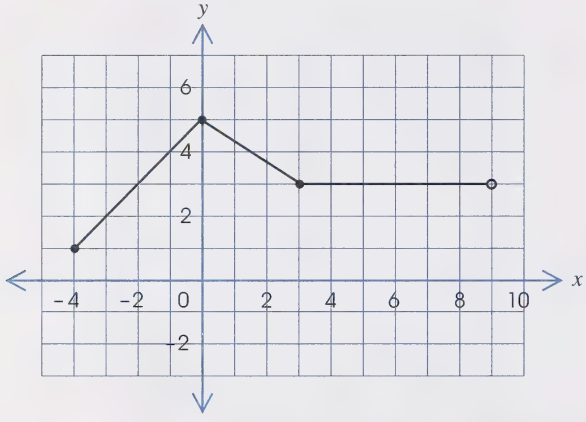


1. Graph the following piecewise functions:

a. $y = x^2 - x$, when $x < 1$
$y = x - 1$, when $1 \le x \le 3$
$y = 11 - x^2$, when $x > 3$

b. $y = x^3$, when $-2 \le x \le 0$
$y = x$, when $0 < x \le 4$
$y = 8 - x$, when $4 < x \le 8$

2. Use the graph at the right to define a piecewise function.



Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

Conclusion

In this section, you identified relations and a special class of relations called functions. You used these relations to model real-world data. You described relations and functions using ordered pairs, a rule in word or equation form, and a graph. To graph both relations and functions, you established tables of values to plot points on a grid or used some type of technology—a computer spreadsheet program and/or a graphing calculator—to assist you. From their graphs, you determined the domain and range and classified relations and functions as linear or non-linear.

When working with functions, you used function notation to evaluate and represent them. You investigated linear functions, discovering that direct and partial variations were two examples of linear functions. You then related these linear functions—in terms of intercepts, slope, domain, and range—to the linear equations you worked with in Section 1.

Wildflowers are part of an ecosystem that involves innumerable relationships. Ecologists study these relationships and advise government and industry on ways of preserving that delicate balance for future generations. The mathematics of relations and functions is one of the essential tools in the study of patterns and relationships.

Assignment

Turn to Assignment Booklet 2B and complete the assignment for Section 2.

Module Summary

In Section 1 of this module, you examined the basic structures of coordinate geometry. You investigated the distance, midpoint, and slope formulas; the techniques for finding the equations of lines; and the procedures for graphing linear equations, using both paper and pencil and a graphing calculator.

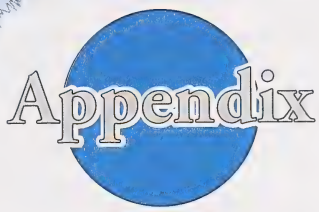
The linear equation is but one example of a relation. In Section 2, you expanded your study of relations, both linear and non-linear. You discovered that some of these relations can be classified as functions and that, for functions, a special notation may be used. Throughout your investigations, you used relations, functions, and their graphs to model real-world data.

The mathematics of relations and functions and the supporting processes of coordinate geometry can be compared to the techniques mimes use to communicate to their audience. Both use symbols, relationships, and graphic images to communicate.

THE EDMONTON JOURNAL

Final Module Assignment

Turn to Assignment Booklet 2B and complete the final module assignment.



Glossary
Suggested Answers

Glossary

absolute value: the distance a real number, x, lies from the origin on the number line, denoted by $|x|$

collinear points: points that lie in the same straight line

direct variation: a linear function of the form $y = kx$

domain: the set of first elements of all the ordered pairs of a relation

extrapolate: to use the graph of a relation to predict values beyond the given ordered pairs

function: a relation where no two distinct ordered pairs have the same first element

function notation: a way of writing a function, f, namely $y = f(x)$

interpolate: to use the graph of a relation to predict values between the given ordered pairs

linear equation: the equation of a line, where each term in the equation is either a constant or of degree 1

linear relation: a relation between two values whose graph is along a straight line

linear function: a function of the form $f(x) = mx + b$

oblique: neither horizontal nor vertical

partial variation: a linear function of the form $y = mx + b$, where $b \neq 0$

point-slope form: the form of an equation of a line based on the slope and a point on the line

$$y - y_1 = m(x - x_1)$$

range: the set of second elements of all the ordered pairs of a relation

relation: a set of ordered pairs

rise: the vertical displacement between two points

run: the horizontal displacement between two points

slope: the ratio of rise to run that measures the steepness of a line

slope and y-intercept form of a linear equation: a linear equation written in the form $y = mx + b$, where m is the slope and b is the y-intercept

standard form of a linear equation: a linear equation written in the form of $Ax + By + C = 0$, where A, B, and C are integers (if possible) and A is positive

vertical line test: a test for determining whether a given graph represents a function

For example, if a vertical line passes through more than one point of a given graph, the graph does not represent a function.

x-intercept: the x-coordinate of the point where a line or curve intersects the x-axis

y-intercept: the y-coordinate of the point where a line or curve intersects the y-axis

Suggested Answers

Section 1: Activity 1

1. a. Textbook questions 31 to 39 of "Practice," p. 8

31. $|-6|=6$

32. $|2-4|=|-2|$
$=2$

33. $|4|=4$

34. $\left|-\frac{2}{3}\right|=\frac{2}{3}$

35. $\left|\frac{1}{2}-\frac{3}{4}\right|=\left|\frac{2}{4}-\frac{3}{4}\right|$
$=\left|-\frac{1}{4}\right|$
$=\frac{1}{4}$

36. $|1.2-1.5|=|-0.3|$
$=0.3$

37. $|-1|+|3|=1+3$
$=4$

38. $|5|-|-2|=5-(+2)$
$=3$

39. $|-3|-|-4|=3-(+4)$
$=-1$

b. Textbook questions 66.c. and 66.f. of "Applications and Problem Solving," p. 9

66. c. This statement is never true. If the irrational number is positive, the absolute value is the irrational number itself. If the irrational number is negative, the absolute value is simply the irrational number changed from negative to positive.

f. This statement is sometimes true. Since 0 is a real number and $|0|=0$, it is neither a positive nor negative number.

2. a. Textbook questions 1 to 5 of Investigation 1, "Lengths on a Number Line," p. 252

1. 7

2. a. $|4-(-3)|=|4+3|$
$=|7|$
$=7$

b. $|-3-4|=|-7|$
$=7$

c. The expressions in 2.a. and 2.b. represent the distance between A and B.

Section 1: Activity 1 (continued)

3. No, the distance between points is always non-negative. That is the reason for using absolute value.

4. a. $|8-3|=|5|$ **or** $|3-8|=|-5|$
$=5$ $=5$

b. $|4-(-5)|=|4+5|$ **or** $|-5-4|=|-9|$
$=|9|$ $=9$
$=9$

c. $|-3-(-9)|=|-3+9|$ **or** $|-9-(-3)|=|-9+3|$
$=|6|$ $=|-6|$
$=6$ $=6$

5. The distance between two points on a number line is the **absolute value** of the difference in their coordinates.

b. Textbook questions 1 to 4 of Investigation 2, "Lengths of Horizontal Line Segments," p. 252

1. Length of $AB=6$
Length of $CD=9$
Length of $EF=7$
Length of $GH=4$

2. The length of a horizontal segment can be found by finding the absolute value of the difference between the x-coordinates of its endpoints.

3. $|x_2-x_1|$ **or** $|x_1-x_2|$

4. a. Length of $ST=|10-4|$
$=|6|$
$=6$

b. Length of $PQ=|2-(-3)|$
$=|2+3|$
$=|5|$
$=5$

c. Length of $KL=|-4-(-7)|$
$=|-4+7|$
$=|3|$
$=3$

d. Length of $VW=|4-(-6)|$
$=|4+6|$
$=|10|$
$=10$

c. Textbook questions 1 to 4 of Investigation 3, "Lengths of Vertical Line Segments," p. 253

1. Length of $AB = 5$

Length of $CD = 8$

Length of $EF = 7$

Length of $GH = 6$

2. The length of a vertical line segment can be found by finding the absolute value of the difference of the y-coordinates of its endpoints.

3. $|y_2 - y_1|$ **or** $|y_1 - y_2|$

4. a. Length of $RS = |1-6|$

$= |-5|$

$= 5$

b. Length of $PQ = |-4-2|$

$= |-6|$

$= 6$

c. Length of $VW = |-3-(-7)|$

$= |-3+7|$

$= |4|$

$= 4$

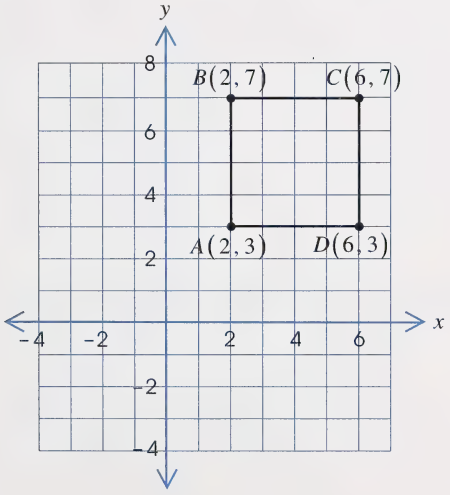
d. Length of $TU = |-3-0|$

$= |-3|$

$= 3$

d. Textbook questions 1.a., 1.c., and 2 of Investigation 4, "Problem Solving," p. 253

1. a.



$ABCD$ is a square.

$BC = AD = |6-2| = 4$

$AB = CD = |7-3| = 4$

Perimeter of $ABCD = 2(4) + 2(4)$

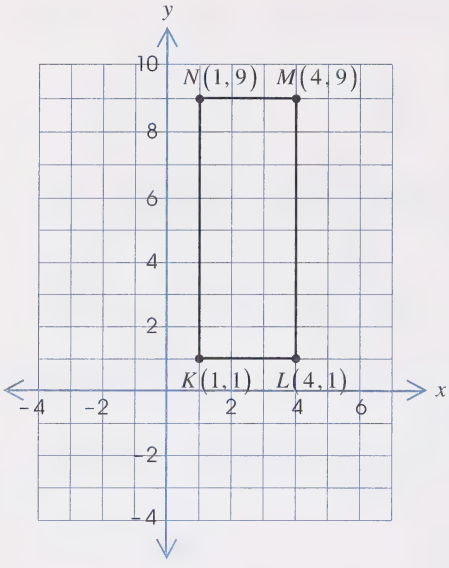
$= 16$ units

Area of $ABCD = 4(4)$

$= 16$ square units

Section 1: Activity 1 (continued)

c.



KLMN is a rectangle.

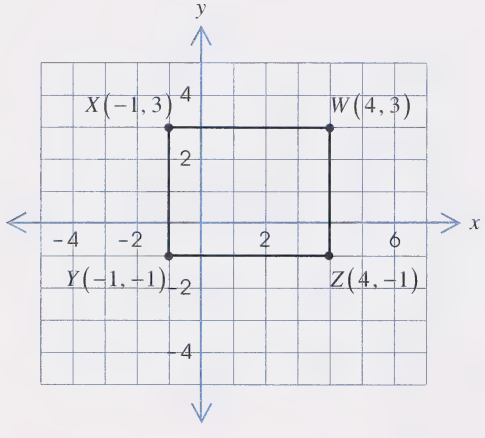
$KL = MN = |4 - 1| = 3$

$LM = KN = |9 - 1| = 8$

Perimeter of $KLMN = 2(3) + 2(8)$
$= 22$ units

Area of $KLMN = 3(8)$
$= 24$ square units

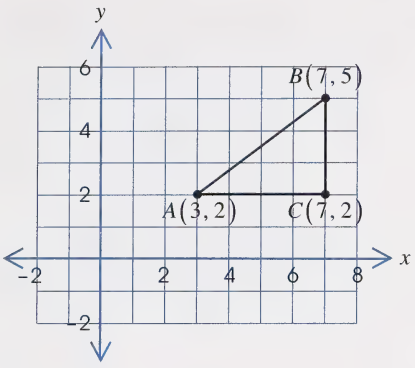
2.



Therefore, the coordinates of Z are $(4, -1)$.

3. a. Textbook questions a. to d. of "Explore: Develop a Method," p. 254

a.



b. Refer to the answer to question a.

c. $C(7,2)$

d. $AC = |7-3|$
$= |4|$
$= 4$

$BC = |2-5|$
$= |-3|$
$= 3$

b. Textbook questions 1 to 6 of "Inquire," p. 255

1. $(AB)^2 = (AC)^2 + (BC)^2$
$(AB)^2 = 4^2 + 3^2$
$(AB)^2 = 16 + 9$
$(AB)^2 = 25$
$AB = 5$

2. The length of AC can be determined by finding the absolute value of the difference between the x-coordinates of A and C.

$AC = |7-3|$
$= |4|$
$= 4$

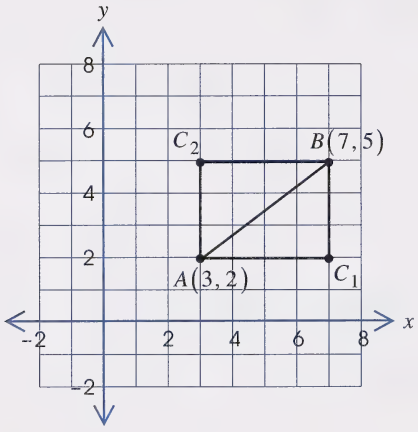
Section 1: Activity 1 (continued)

3. The length of BC can be determined by finding the absolute value of the difference between the y-coordinates of B and C.

$$\begin{aligned} BC &= |2-5| \\ &= |-3| \\ &= 3 \end{aligned}$$

4. The length of AB would not change because the right triangle formed by locating point C above point A would be congruent (same shape and size) to the right triangle formed by locating point C below point B.

$$\Delta ABC_1 \cong \Delta ABC_2$$



5. distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

6. a. Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (10, 9)$.

$$\begin{aligned}
\therefore CD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(10-2)^2 + (9-3)^2} \\
&= \sqrt{8^2 + 6^2} \\
&= \sqrt{64 + 36} \\
&= \sqrt{100} \\
&= 10
\end{aligned}$$

b. Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (7, 1)$.

$$\begin{aligned}
\therefore EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(7-2)^2 + (1-4)^2} \\
&= \sqrt{5^2 + (-3)^2} \\
&= \sqrt{25 + 9} \\
&= \sqrt{34} \\
&\doteq 5.8
\end{aligned}$$

c. Let $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (4, 5)$.

$$\begin{aligned}
\therefore GH &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{[4-(-3)]^2 + (5-1)^2} \\
&= \sqrt{(4+3)^2 + 4^2} \\
&= \sqrt{7^2 + 4^2} \\
&= \sqrt{49 + 16} \\
&= \sqrt{65} \\
&\doteq 8.1
\end{aligned}$$

Section 1: Activity 1 (continued)

4. a. Textbook questions 1 to 6 of "Practice," p. 256

1. Let $(x_1, y_1) = (2,1)$ and $(x_2, y_2) = (3,5)$.

$$\begin{aligned}\therefore \text{ distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-2)^2 + (5-1)^2} \\ &= \sqrt{1^2 + 4^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17}\end{aligned}$$

2. Let $(x_1, y_1) = (3,-5)$ and $(x_2, y_2) = (-6,7)$.

$$\begin{aligned}\therefore \text{ distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-6-3)^2 + [7-(-5)]^2} \\ &= \sqrt{(-9)^2 + (7+5)^2} \\ &= \sqrt{81+12^2} \\ &= \sqrt{81+144} \\ &= \sqrt{225} \\ &= 15\end{aligned}$$

3. Let $(x_1, y_1) = (3,0)$ and $(x_2, y_2) = (4,-1)$.

$$\begin{aligned}\therefore \text{ distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-3)^2 + (-1-0)^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2}\end{aligned}$$

4. Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (-6, -3)$.

$$\begin{aligned}
\therefore \text{ distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{[-6-(-1)]^2 + (-3-2)^2} \\
&= \sqrt{(-6+1)^2 + (-5)^2} \\
&= \sqrt{(-5)^2 + 25} \\
&= \sqrt{25+25} \\
&= \sqrt{50} \\
&= \sqrt{25 \times 2} \\
&= \sqrt{25} \times \sqrt{2} \\
&= 5\sqrt{2}
\end{aligned}$$

5. Let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (2, 9)$.

$$\begin{aligned}
\therefore \text{ distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(2-2)^2 + (9-1)^2} \\
&= \sqrt{0^2 + 8^2} \\
&= \sqrt{0+64} \\
&= \sqrt{64} \\
&= 8
\end{aligned}$$

6. Let $(x_1, y_1) = (4, -7)$ and $(x_2, y_2) = (11, -7)$.

$$\begin{aligned}
\therefore \text{ distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(11-4)^2 + [-7-(-7)]^2} \\
&= \sqrt{(7)^2 + (-7+7)^2} \\
&= \sqrt{49+0} \\
&= \sqrt{49} \\
&= 7
\end{aligned}$$

Section 1: Activity 1 (continued)

b. Textbook questions 13.a., 14, 20.a., and 24 of "Applications and Problem Solving," p. 257

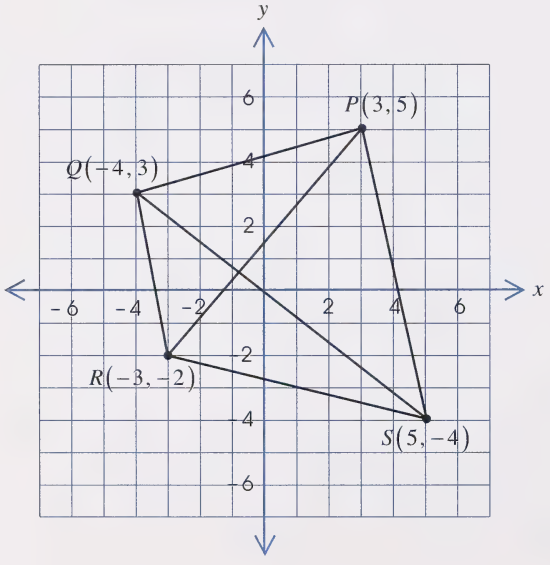
13. a. Ottawa is $(2241, 4378)$ and Toronto is $(2488, 4981)$.

Let $(x_1, y_1) = (2241, 4378)$ and $(x_2, y_2) = (2488, 4981)$.

$$\therefore \text{ distance (in grid units)} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2488 - 2241)^2 + (4981 - 4378)^2}$$
$$= \sqrt{(247)^2 + (603)^2}$$
$$= \sqrt{61\,009 + 363\,609}$$
$$= \sqrt{424\,618}$$

Now, each unit of length on the grid is 0.51 km. Therefore, the distance is $\sqrt{424\,618} \times 0.51 \doteq 332$ km.

14.



To find the length of diagonal *PR*, let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (-3, -2)$.

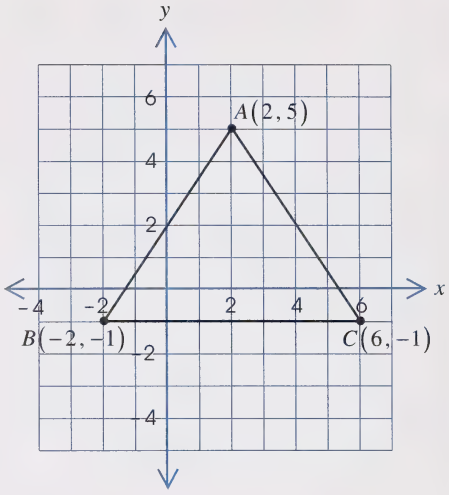
$$\begin{aligned} PR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-3)^2 + (-2-5)^2} \\ &= \sqrt{(-6)^2 + (-7)^2} \\ &= \sqrt{36+49} \\ &= \sqrt{85} \\ &\doteq 9.2 \end{aligned}$$

To find the length of diagonal *QS*, let $(x_1, y_1) = (-4, 3)$ and $(x_2, y_2) = (5, -4)$.

$$\begin{aligned} QS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[5-(-4)]^2 + (-4-3)^2} \\ &= \sqrt{(5+4)^2 + (-7)^2} \\ &= \sqrt{9^2 + 49} \\ &= \sqrt{81+49} \\ &= \sqrt{130} \\ &\doteq 11.4 \end{aligned}$$

Section 1: Activity 1 (continued)

20. a. To find the length of AB, let $(x_1, y_1) = (2, 5)$ and $(x_2, y_2) = (-2, -1)$.



$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-2)^2 + (-1-5)^2} \\ &= \sqrt{(-4)^2 + (-6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= \sqrt{4 \times 13} \\ &= \sqrt{4} \times \sqrt{13} \\ &= 2\sqrt{13} \end{aligned}$$

To find the length of AC, let $(x_1 - y_1) = (2, 5)$ and $(x_2, y_2) = (6, -1)$.

$$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6-2)^2 + (-1-5)^2} \\ &= \sqrt{4^2 + (-6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= \sqrt{4 \times 13} \\ &= \sqrt{4} \times \sqrt{13} \\ &= 2\sqrt{13} \end{aligned}$$

Find the length of BC.

$$\begin{aligned} BC &= |6 - (-2)| \\ &= |6 + 2| \\ &= 8 \end{aligned}$$

$\therefore$ Perimeter $= AB + AC + BC$

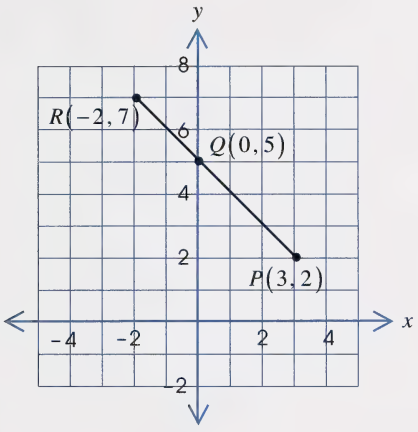
$= 2\sqrt{13} + 2\sqrt{13} + 8$

$= 4\sqrt{13} + 8$

$\doteq 22.4$

Therefore, ΔABC is an isosceles triangle with a perimeter of approximately 22.4 units.

24.



First, find the lengths of PQ and QR; then find the length of PR. If $PQ + QR = PR$, the points are collinear.

For PQ, let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (0, 5)$.

$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-3)^2 + (5-2)^2}$

$= \sqrt{(-3)^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= \sqrt{9 \times 2}$

$= \sqrt{9} \times \sqrt{2}$

$= 3\sqrt{2}$

Section 1: Activity 1 (continued)

For QR, let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (-2, 7)$.

$$\begin{aligned}
\therefore QR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(-2-0)^2 + (7-5)^2} \\
&= \sqrt{(-2)^2 + 2^2} \\
&= \sqrt{4+4} \\
&= \sqrt{8} \\
&= \sqrt{4 \times 2} \\
&= \sqrt{4} \times \sqrt{2} \\
&= 2\sqrt{2}
\end{aligned}$$

For PR, let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (-2, 7)$.

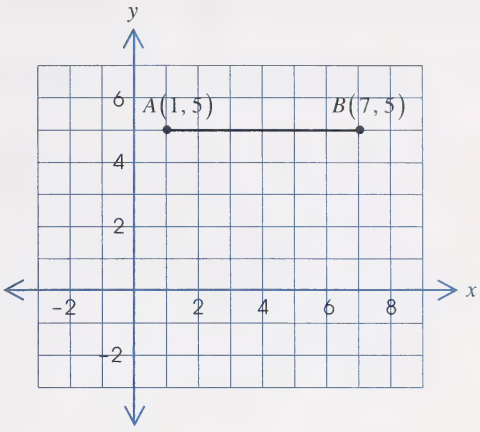
$$\begin{aligned}
\therefore PR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(-2-3)^2 + (7-2)^2} \\
&= \sqrt{(-5)^2 + (5^2)} \\
&= \sqrt{25+25} \\
&= \sqrt{50} \\
&= \sqrt{25 \times 2} \\
&= \sqrt{25} \times \sqrt{2} \\
&= 5\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
PQ + QR &= 3\sqrt{2} + 2\sqrt{2} \\
&= 5\sqrt{2} \quad \longleftarrow \text{This is equal to } PR.
\end{aligned}$$

Therefore, P, Q, and R are collinear.

5. a. Using the general distance formula,
let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (7, 5)$.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(7-1)^2 + (5-5)^2} \\ &= \sqrt{6^2 + 0^2} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

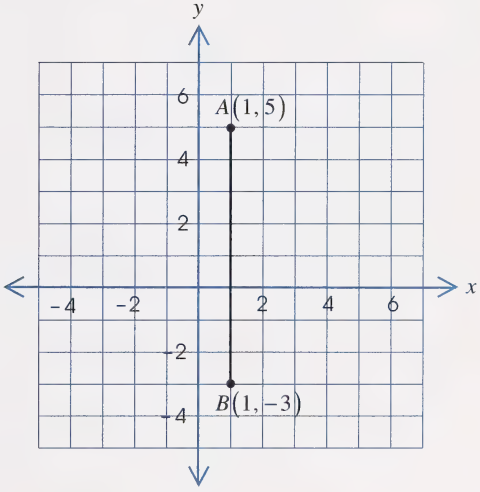


The points form a horizontal segment. Thus, use the horizontal distance formula.

$$\begin{aligned} AB &= |x_2 - x_1| \\ &= |7-1| \\ &= 6 \end{aligned}$$

b. Using the general distance formula,
let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (1, -3)$.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-1)^2 + (-3-5)^2} \\ &= \sqrt{0^2 + (-8)^2} \\ &= \sqrt{0+64} \\ &= 8 \end{aligned}$$



The points form a vertical segment. Thus, use the vertical distance formula.

$$\begin{aligned} AB &= |y_2 - y_1| \\ &= |5-(-3)| \\ &= |5+3| \\ &= 8 \end{aligned}$$

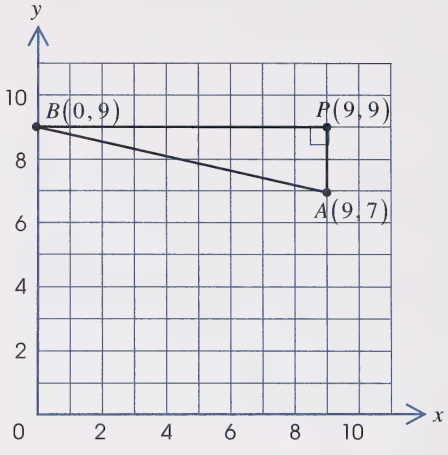
Section 1: Activity 1 (continued)

6. Textbook questions 1.a., 2, and 3 of Investigation 1, "Taxicab Geometry," p. 250

1. a. $d_T = 11$ blocks

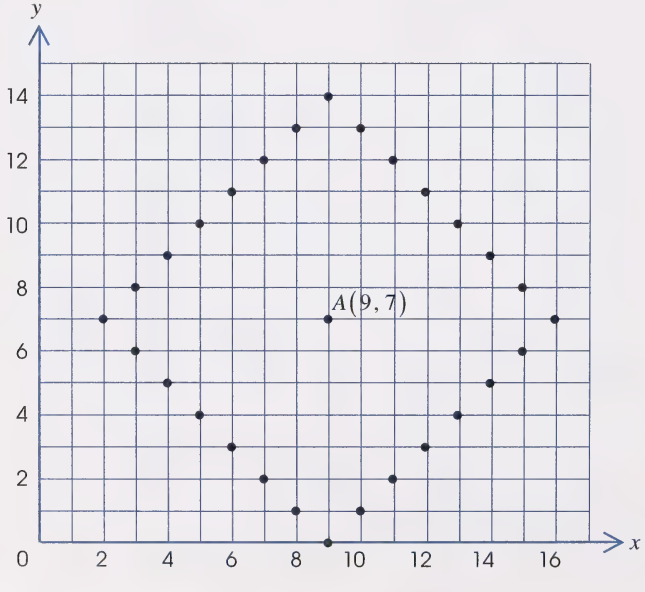
$$AB^2 = BP^2 + AP^2$$
$$AB^2 = 9^2 + 2^2$$
$$AB^2 = 81 + 4$$
$$AB^2 = 85$$
$$AB = \sqrt{85}$$

$\therefore d_E = \sqrt{85}$ blocks



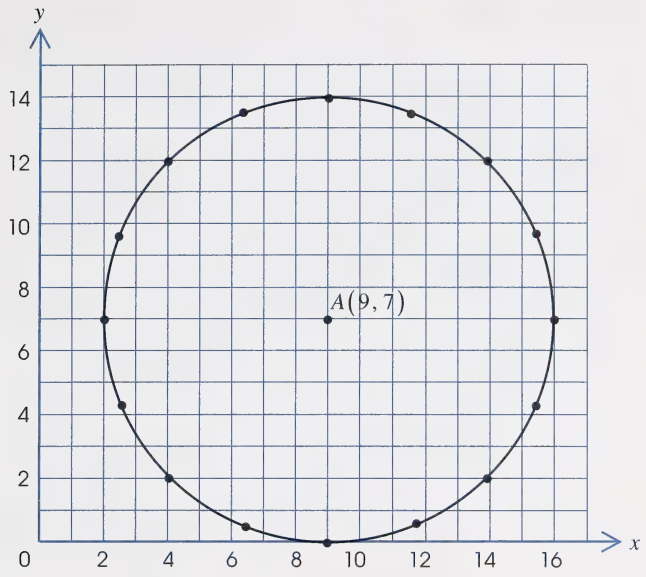
2. $d_T = d_E$ when the Euclidean distance is the length of a horizontal or vertical segment; that is, when the taxicab can get to its destination without turning.

3. a.



The taxicab distances of 7 units from point A form a square.

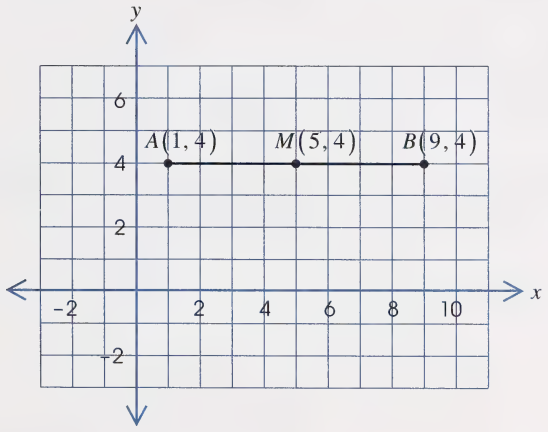
b.



The Euclidean distances of 7 units from point A form a circle, where point A is the centre and the radius is 7 units.

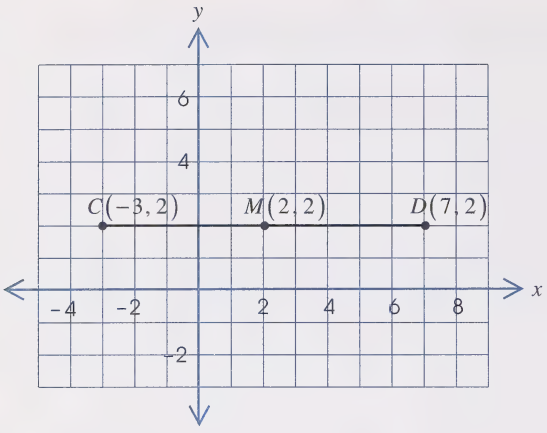
7. a. Textbook questions 1.a. to 1.c., 2, 3.a., 3.c., and 4 of Investigation 1, "Midpoints of Horizontal Line Segments," p. 258

1. a.

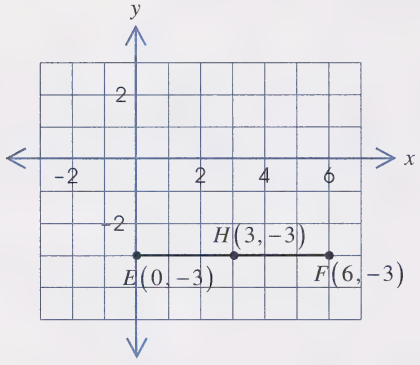


Section 1: Activity 1 (continued)

b.



c.



2. The x-coordinate of the midpoint is the average of the x-coordinates of the endpoints, $\frac{x_1+x_2}{2}$. The y-coordinate of the midpoint is the same as the y-coordinate of the endpoints.

3. **a.** The midpoint of KL is $\left(\frac{2+6}{2}, 3\right) = (4, 3)$.

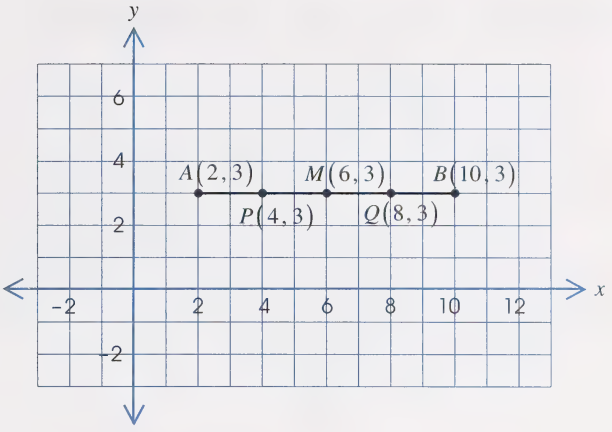
c. The midpoint of RS is $\left(\frac{-7+(-3)}{2}, 5\right) = (-5, 5)$.

4. Let M be the midpoint of AB. Thus, the coordinates of M are $\left(\frac{2+10}{2}, 3\right) = (6, 3)$.

Let P be the midpoint of AM. Thus, the coordinates of P are $\left(\frac{2+6}{2}, 3\right) = (4, 3)$.

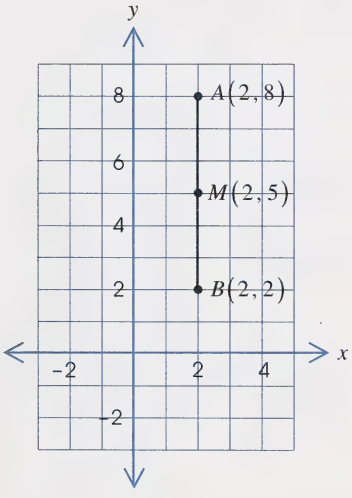
Let Q be the midpoint of BM. Thus, the coordinates of Q are $\left(\frac{10+6}{2}, 3\right) = (8, 3)$.

P, M, and Q divide AB into 4 equal parts.

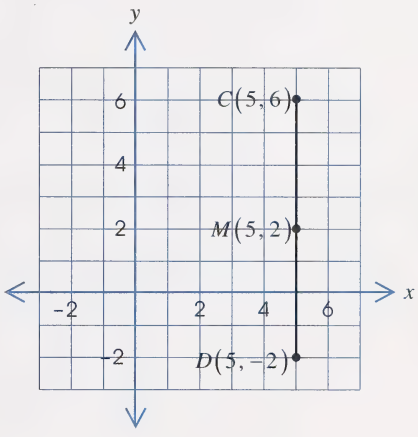


b. Textbook questions 1.a. to 1.c., 2, 3.a., 3.c., and 4 of Investigation 2, "Midpoints of Vertical Line Segments," p. 258

1. a.

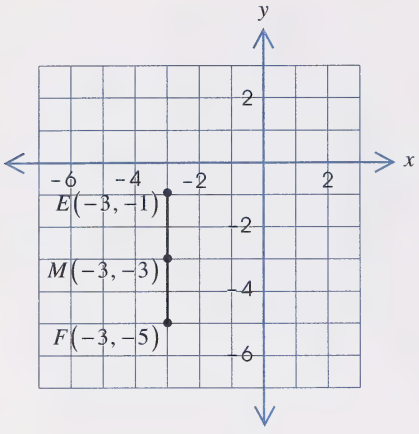


b.



Section 1: Activity 1 (continued)

c.



2. The y-coordinate of the midpoint is the average of the y-coordinates of the endpoints of the vertical segment, $\frac{y_1+y_2}{2}$. The x-coordinate is the same as the x-coordinates of the endpoints.

3. **a.** The midpoint of ST is $\left(4, \frac{7+1}{2}\right) = (4, 4)$.

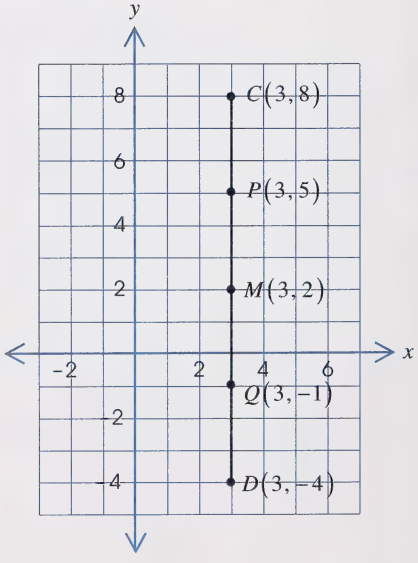
c. The midpoint of VW is $\left(-2, \frac{-4+4}{2}\right) = (-2, 0)$.

4. Let M be the midpoint of CD. Thus, the coordinates of M are $\left(3, \frac{8+(-4)}{2}\right) = (3, 2)$.

Let P be the midpoint of CM. Thus, the coordinates of P are $\left(3, \frac{8+2}{2}\right) = (3, 5)$.

Let Q be the midpoint of DM. Thus, the coordinates of Q are $\left(3, \frac{-4+2}{2}\right) = (3, -1)$.

P, M, and Q divide CD into 4 equal parts.



8. a. Textbook questions a. to c. of "Explore: Use the Diagrams," p. 259

a. $A(1,5)$, $A'(5,1)$, $P(3,3)$

b. $B(1,3)$, $B'(-3,-1)$, $Q(-1,1)$

c. $C(2,5)$, $C'(4,-1)$, $R(3,2)$

b. Textbook questions 1 to 5 of "Inquire," p. 259

1. a. The x-coordinate of P is the average of the x-coordinates of A and A'.

$$\frac{1+5}{2}=3$$

b. The y-coordinate of P is the average of the y-coordinates of A and A'.

$$\frac{5+1}{2}=3$$

2. a. The x-coordinate of Q is the average of the x-coordinates of B and B'.

$$\frac{1+(-3)}{2}=-1$$

b. The y-coordinate of Q is the average of the y-coordinates of B and B'.

$$\frac{3+(-1)}{2}=1$$

3. a. The x-coordinate of R is the average of the x-coordinates of C and C'.

$$\frac{2+4}{2}=3$$

b. The y-coordinate of R is the average of the y-coordinates of C and C'.

$$\frac{5+(-1)}{2}=2$$

4. If (x_1, y_1) and (x_2, y_2) are the endpoints of a segment, the midpoint of the segment is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Section 1: Activity 1 (continued)

5. a. The midpoint of $A(4,9)$ and $B(2,1)$ is $\left(\frac{4+2}{2}, \frac{9+1}{2}\right) = (3,5)$.

b. The midpoint of $C(0,-2)$ and $D(-2,4)$ is $\left(\frac{0+(-2)}{2}, \frac{-2+4}{2}\right) = (-1,1)$.

c. The midpoint of $E(-3,-4)$ and $F(-1,6)$ is $\left(\frac{-3+(-1)}{2}, \frac{-4+6}{2}\right) = (-2,1)$.

9. a. Textbook questions 1, 3, 5, 8, 11, and 12 of "Practice," p. 261

1. Let $(x_1, y_1) = (5,7)$ and $(x_2, y_2) = (3,9)$.

$$\therefore \text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{5+3}{2}, \frac{7+9}{2}\right)$$
$$= (4,8)$$

3. Let $(x_1, y_1) = (-1,0)$ and $(x_2, y_2) = (1,-6)$.

$$\therefore \text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-1+1}{2}, \frac{0+(-6)}{2}\right)$$
$$= (0,-3)$$

5. Let $(x_1, y_1) = (-3,-3)$ and $(x_2, y_2) = (-1,-7)$.

$$\therefore \text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-3+(-1)}{2}, \frac{-3+(-7)}{2}\right)$$
$$= (-2,-5)$$

8. Let $(x_1, y_1) = \left(\frac{1}{2}, \frac{5}{2}\right)$ and $(x_2, y_2) = \left(\frac{3}{2}, \frac{-7}{2}\right)$.

$$\therefore \text{ Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{\frac{1}{2} + \frac{3}{2}}{2}, \frac{\frac{5}{2} + \left(\frac{-7}{2}\right)}{2}\right)$$

$$= \left(\frac{\frac{4}{2}}{2}, \frac{\frac{-2}{2}}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-1}{2}\right)$$

$$= \left(1, -\frac{1}{2}\right)$$

11. Let the coordinates of the missing endpoint be (x_1, y_1). One endpoint is $(3, -5)$, and the midpoint is $(7, 7)$.

$$\therefore \text{ Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(7, 7) = \left(\frac{x_1 + 3}{2}, \frac{y_1 + (-5)}{2}\right)$$

Solve for x_1.

$$\frac{x_1 + 3}{2} = 7$$

$$2\left(\frac{x_1 + 3}{2}\right) = 2(7)$$

$$x_1 + 3 = 14$$

$$x_1 = 11$$

Solve for y_1.

$$\frac{y_1 + (-5)}{2} = 7$$

$$2\left(\frac{y_1 + (-5)}{2}\right) = 2(7)$$

$$y_1 - 5 = 14$$

$$y_1 = 19$$

The missing endpoint is $(11, 19)$.

Section 1: Activity 1 (continued)

12. Let the coordinates of the missing endpoint be (x_1, y_1). One endpoint is $(6, 10)$, and the midpoint is $(0, 0)$.

$$\therefore \text{ Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(0,0) = \left(\frac{x_1 + 6}{2}, \frac{y_1 + 10}{2}\right)$$

Solve for x_1.

$$\frac{x_1 + 6}{2} = 0$$

$$2\left(\frac{x_1 + 6}{2}\right) = 2(0)$$

$$x_1 + 6 = 0$$

$$x_1 = -6$$

Solve for y_1.

$$\frac{y_1 + 10}{2} = 0$$

$$2\left(\frac{y_1 + 10}{2}\right) = 2(0)$$

$$y_1 + 10 = 0$$

$$y_1 = -10$$

The missing endpoint is $(-6, -10)$.

b. Textbook questions 21, 22, 25.d., 29, and 36.a. of "Applications and Problem Solving," pp. 261 and 262

21. a. Let $(x_1, y_1) = (4, 4)$ and $(x_2, y_2) = (-2, 0)$.

$$\therefore A\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = A\left(\frac{4 + (-2)}{2}, \frac{4 + 0}{2}\right)$$

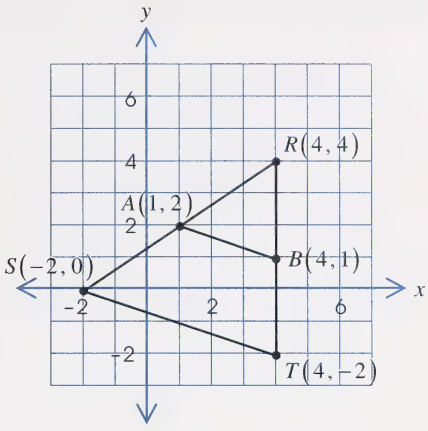
$$= A(1, 2)$$

b. Let $(x_1, y_1) = (4, 4)$ and $(x_2, y_2) = (4, -2)$.

$$\therefore B\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = B\left(\frac{4 + 4}{2}, \frac{4 + (-2)}{2}\right)$$

$$= B(4, 1)$$

c. Find the length of AB. Let $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (4, 1)$.



$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-1)^2 + (1-2)^2} \\ &= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{9+1} \\ &= \sqrt{10} \end{aligned}$$

Now, find the length of ST. Let $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (4, -2)$.

$$\begin{aligned} ST &= \sqrt{[4-(-2)]^2 + (-2-0)^2} \\ &= \sqrt{(4+2)^2 + (-2)^2} \\ &= \sqrt{6^2 + 4} \\ &= \sqrt{36+4} \\ &= \sqrt{40} \\ &= \sqrt{4 \times 10} \\ &= \sqrt{4} \times \sqrt{10} \\ &= 2\sqrt{10} \end{aligned}$$

Therefore, $AB = \frac{1}{2} ST$.

Section 1: Activity 1 (continued)

22. Let the coordinates of the missing endpoint of the diameter be (x_1, y_1). The centre of the circle, $(-1,-3)$, is the midpoint of the diameter, and one endpoint of the diameter is $(-3,0)$.

$$\therefore \text{ Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$(-1,-3) = \left(\frac{x_1+(-3)}{2}, \frac{y_1+0}{2}\right)$$

$$= \left(\frac{x_1-3}{2}, \frac{y_1}{2}\right)$$

Solve for x_1.

$$\frac{x_1-3}{2} = -1$$

$$2\left(\frac{x_1-3}{2}\right) = 2(-1)$$

$$x_1 - 3 = -2$$

$$x_1 = 1$$

Solve for y_1.

$$\frac{y_1}{2} = -3$$

$$2\left(\frac{y_1}{2}\right) = 2(-3)$$

$$y_1 = -6$$

Therefore, the other endpoint of the diameter is $(1,-6)$.

25. d. Let $(x_1, y_1) = (2t, 3t+1)$ and $(x_2, y_2) = (-4t, 1-t)$.

$$\therefore \text{ Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{2t+(-4t)}{2}, \frac{(3t+1)+(1-t)}{2}\right)$$

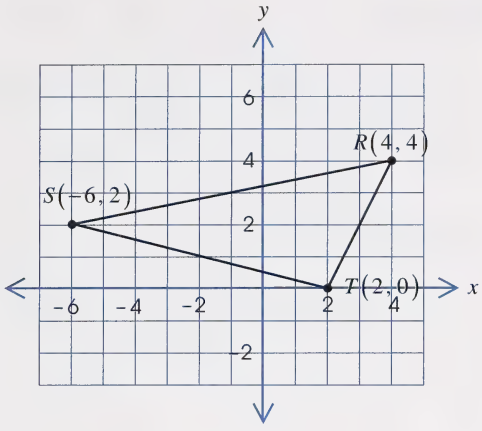
$$= \left(\frac{2t-4t}{2}, \frac{3t+1+1-t}{2}\right)$$

$$= \left(\frac{-2t}{2}, \frac{2t+2}{2}\right)$$

$$= \left(-t, \frac{2(t+1)}{2}\right)$$

$$= (-t, t+1)$$

29. a.



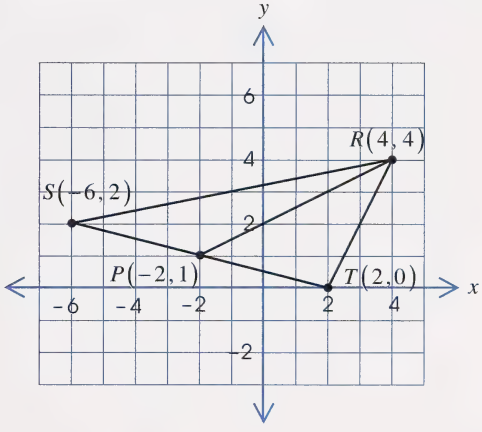
b. To find the length of the median from R, first find the midpoint of side ST. Let P be the midpoint of ST, $(x_1, y_1) = (-6, 2)$, and $(x_2, y_2) = (2, 0)$.

A median is a line segment that joins a vertex of a triangle to the midpoint of the opposite side.

$$P\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = P\left(\frac{-6+2}{2}, \frac{2+0}{2}\right)$$
$$= P(-2, 1)$$

Next, determine the distance from R to P. Let $(x_1, y_1) = (4, 4)$ and $(x_2, y_2) = (-2, 1)$.

$$RP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2-4)^2 + (1-4)^2}$$
$$= \sqrt{(-6)^2 + (-3)^2}$$
$$= \sqrt{36+9}$$
$$= \sqrt{45}$$
$$= \sqrt{9 \times 5}$$
$$= \sqrt{9} \times \sqrt{5}$$
$$= 3\sqrt{5}$$



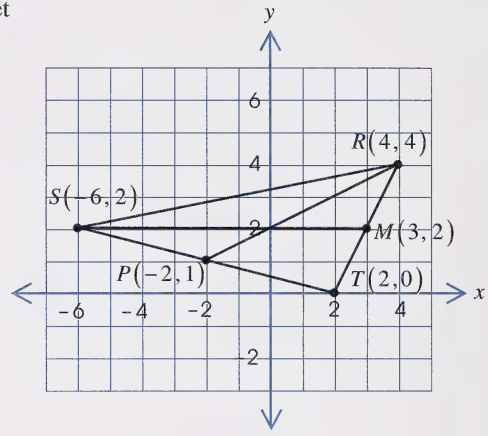
Thus, the length of the median from R is $3\sqrt{5}$.

Section 1: Activity 1 (continued)

To find the length of the median from S, determine the midpoint of side RT. Let M be the midpoint of RT, $(x_1, y_1) = (4, 4)$, and $(x_2, y_2) = (2, 0)$.

$$\therefore M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{4+2}{2}, \frac{4+0}{2}\right)$$
$$= M(3, 2)$$

Now, determine the distance from S to M. Let $(x_1, y_1) = (-6, 2)$ and $(x_2, y_2) = (3, 2)$.



$$SM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[3-(-6)]^2 + (2-2)^2}$$
$$= \sqrt{(3+6)^2 + 0^2}$$
$$= \sqrt{9^2 + 0}$$
$$= \sqrt{81}$$
$$= 9$$

Thus, the length of the median from S is 9.

Note: The length of the median from S may also have been found using $|x_2 - x_1|$, since SM is a horizontal segment.

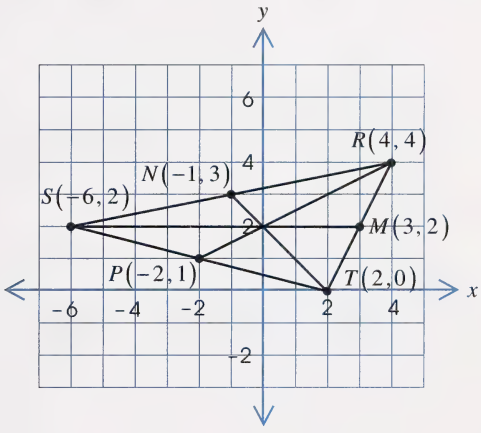
$$SM = |x_2 - x_1|$$
$$= |3-(-6)|$$
$$= |3+6|$$
$$= |9|$$
$$= 9$$

Finally, to determine the length of the median from T, first find the midpoint of RS. Let N be the midpoint of RS, $(x_1, y_1) = (4, 4)$, and $(x_2, y_2) = (-6, 2)$.

$$N\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = N\left(\frac{4 + (-6)}{2}, \frac{4 + 2}{2}\right)$$
$$= N(-1, 3)$$

Next, determine the distance between T and N. Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (-1, 3)$.

$$TN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1-2)^2 + (3-0)^2}$$
$$= \sqrt{(-3)^2 + 3^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= \sqrt{9 \times 2}$$
$$= \sqrt{9} \times \sqrt{2}$$
$$= 3\sqrt{2}$$



Thus, the length of the median from T is $3\sqrt{2}$.

36. a. To find AM, let $(x_1, y_1) = (1, -2)$ and $(x_2, y_2) = (3, 2)$.

$$AM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-1)^2 + [2-(-2)]^2}$$
$$= \sqrt{2^2 + 4^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= \sqrt{4 \times 5}$$
$$= \sqrt{4} \times \sqrt{5}$$
$$= 2\sqrt{5}$$

Section 1: Activity 1 (continued)

Next, find *MB*. Let $(x_1, y_1) = (5, 6)$ and $(x_2, y_2) = (3, 2)$.

$$\begin{aligned} BM &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-5)^2 + (2-6)^2} \\ &= \sqrt{(-2)^2 + (-4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= \sqrt{4 \times 5} \\ &= \sqrt{4} \times \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

Therefore, $AM = BM$.

10. Textbook questions 1 to 6 of "Mental Math: Order of Operations," p. 251

1. 6 **2.** 9 **3.** 3

4. 2 **5.** -3 **6.** -5

Section 1: Activity 2

1. a. Textbook questions a. to c. of "Explore: Develop a Method," p. 264

a. rise = 5 and run = 5

$$\therefore \text{ slope} = \frac{\text{rise}}{\text{run}} = \frac{5}{5} = 1$$

b. rise = 6 and run = 3

$$\therefore \text{ slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$$

c. rise = 2 and run = 6

$$\therefore \text{ slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

b. Textbook questions 1 to 6 of "Inquire," p. 264

1. Segment *CD* is the steepest.

2. Segment *CD* has the greatest slope.

3. The rise of each line segment can be calculated by subtracting the *y*-coordinate of the endpoint on the left from the *y*-coordinate of the endpoint on the right.

$y_2 - y_1$

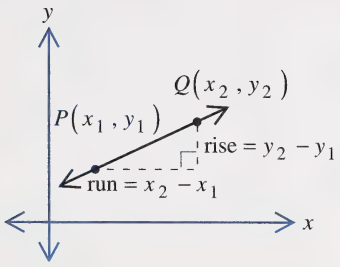
from right-hand endpoint — from left-hand endpoint

4. The run of each line segment can be calculated by subtracting the *x*-coordinate of the left-hand endpoint from the *x*-coordinate of the right-hand endpoint.

$x_2 - x_1$

from right-hand endpoint — from left-hand endpoint

5.



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

6. slope of the Great Pyramid $= \frac{\text{rise}}{\text{run}}$
$= \frac{5}{4}$

slope of Russian Hill $= \frac{\text{rise}}{\text{run}}$
$= \frac{3}{10}$

The face of the Great Pyramid has the greater slope.

Section 1: Activity 2 (continued)

2. a. Textbook questions 1 to 4, 8, 10, 12, and 16 of "Practice," p. 267

1. a. positive **b.** rise = 3 **c.** run = 3 **d.** $\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{3}{3}$

$= 1$

2. a. zero **b.** rise = 0 **c.** run = 4 **d.** $\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{0}{4}$

$= 0$

3. a. negative **b.** rise = -4 **c.** run = 5 **d.** $\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{5}$

$= -\frac{4}{5}$

4. a. undefined **b.** rise = 7 **c.** run = 0 **d.** $\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{7}{0}$

= undefined

8. Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-2, -4)$.

$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-4 - 0}{-2 - 0}$

$= \frac{-4}{-2}$

$= 2$

10. Let $(x_1, y_1) = (-4, 5)$ and $(x_2, y_2) = (6, 5)$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5-5}{6-(-4)}$$
$$= \frac{0}{6+4}$$
$$= 0$$

12. Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (4, 0)$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0-6}{4-0}$$
$$= \frac{-6}{4}$$
$$= \frac{-3}{2}$$

16. Let $(x_1, y_1) = (-8, -7)$ and $(x_2, y_2) = (-4, -3)$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-3-(-7)}{-4-(-8)}$$
$$= \frac{-3+7}{-4+8}$$
$$= \frac{4}{4}$$
$$= 1$$

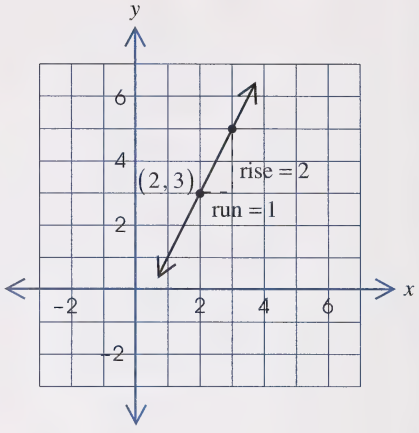
Section 1: Activity 2 (continued)

b. Textbook questions 30.a., 30.e., 30.f., 32.e., and 32.f. of "Applications and Problem Solving," p. 268

30. a. First, plot point $(2,3)$.

Now, use the slope to locate a second point.
Because $m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$, count 1 unit to the right and 2 units up from point $(2,3)$.

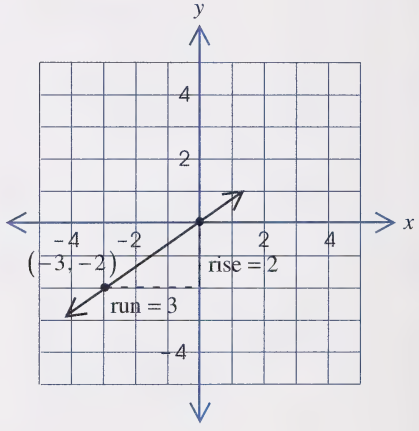
Finally, draw a line through the two points.



e. Plot point $(-3,-2)$.

Because $m = \frac{\text{rise}}{\text{run}} = \frac{2}{3}$, count 3 units to the right and 2 units up from $(-3,-2)$ to locate a second point.

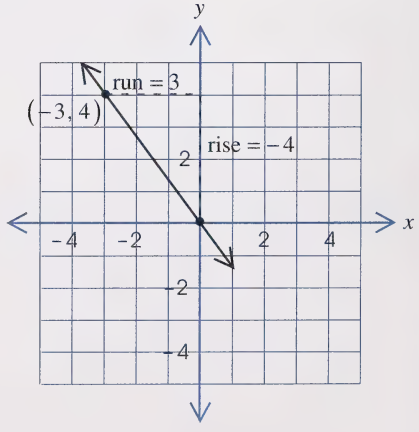
Finally, draw a line through the two points.



f. Plot point $(-3,4)$.

Because $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{3}$, count 3 units to the right and 4 units down from $(-3,4)$ to locate a second point.

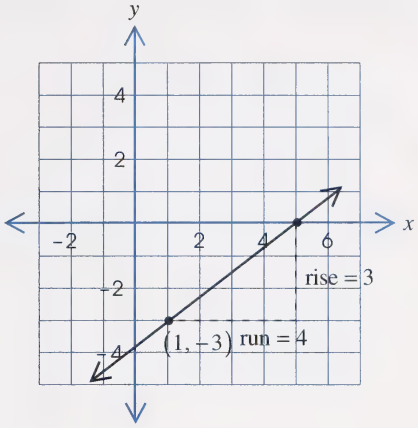
Finally, draw a line through the two points.



32. e. Plot point $(1, -3)$.

Because $m = \frac{\text{rise}}{\text{run}} = \frac{3}{4}$, count 4 units to the right and 3 units up from $(1, -3)$.

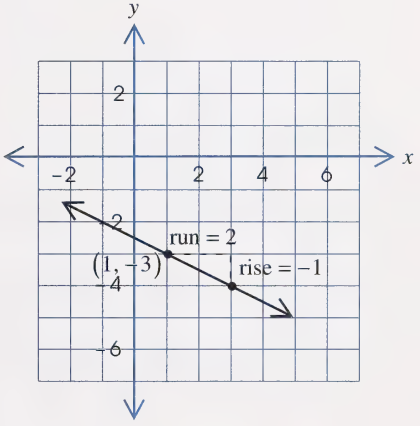
Thus, the second point is $(5, 0)$.



f. Plot point $(1, -3)$.

Because $m = \frac{\text{rise}}{\text{run}} = \frac{-1}{2}$, count 2 units to the right and 1 unit down from $(1, -3)$.

Thus, the second point is $(3, -4)$.



3. a. Textbook questions 24 and 25 of "Practice," p. 268

24. Let $(x_1, y_1) = (2, k)$, $(x_2, y_2) = (4, 1)$, and $m = 3$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$3 = \frac{1-k}{4-2}$$

$$3 = \frac{1-k}{2}$$

$$6 = 1-k$$

$$5 = -k$$

$$k = -5$$

Section 1: Activity 2 (continued)

25. Let $(x_1, y_1) = (t, -1)$, $(x_2, y_2) = (-4, 9)$, and $m = -2$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-2 = \frac{9-(-1)}{-4-t}$$

$$-2 = \frac{9+1}{-4-t}$$

$$-2 = \frac{10}{-4-t}$$

$$-2(-4-t) = 10$$

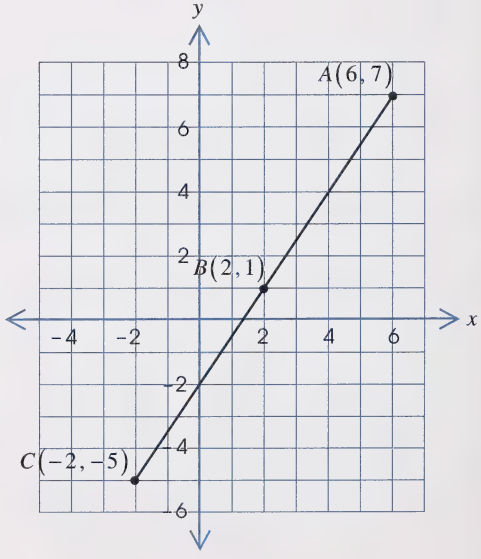
$$8+2t = 10$$

$$2t = 2$$

$$t = 1$$

b. Textbook questions 42.a. and 42.b. of "Applications and Problem Solving," p. 269

42. a. Points are collinear if a single line can be drawn through them. You will recall that the slope of a line is constant. Therefore, to show that A, B, and C are collinear, find out if $m_{AB} = m_{BC}$.



First, find the slope of AB. Let $(x_1, y_1) = (6, 7)$ and $(x_2, y_2) = (2, 1)$.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-7}{2-6}$$

$$= \frac{-6}{-4}$$

$$= \frac{3}{2}$$

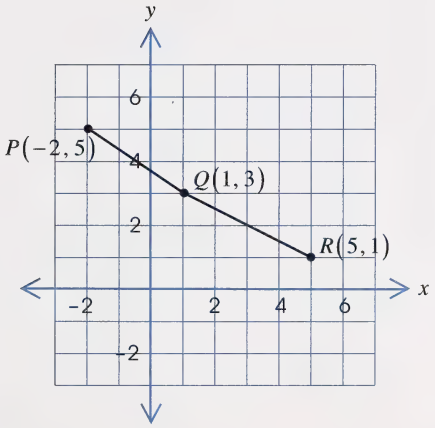
Now, find the slope of BC. Let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (-2, -5)$.

$$\begin{aligned} m_{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5-1}{-2-2} \\ &= \frac{-6}{-4} \\ &= \frac{3}{2} \end{aligned}$$

Because $m_{AB} = m_{BC} = \frac{3}{2}$, points A, B, and C are collinear.

b. $$\begin{aligned} m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-5}{1-(-2)} \\ &= \frac{-2}{3} \\ &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} m_{QR} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-3}{5-1} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2} \end{aligned}$$



Because $m_{PQ} \neq m_{QR}$, points P, Q, and R are not collinear.

Section 1: Activity 2 (continued)

4. a. Textbook question "Explore: Interpret the Graph," p. 270

$$\begin{aligned} m &= \frac{1500-500}{312-108} \\ &= \frac{1000}{204} \\ &\doteq 4.9 \end{aligned}$$

b. Textbook questions 1 to 3 of "Inquire," p. 270

1. The units for speed should be m/s because the distance is expressed in metres and the time is given in seconds.

2. a. The speed for the first 500 m should be slower than the last 1000, because the rowers start from rest in the first 500 metres, reach their top speed, and then sprint to the finish.

b. The average speed for the first 500 metres is $\frac{500 \text{ m}}{108 \text{ s}} \doteq 4.6 \text{ m/s}$.

c. A slope that represents a higher speed is steeper.

3. a. The point $(k, 1250)$ lies on the graph between $(108, 500)$ and $(312, 1500)$. Therefore, k is between 108 s and 312 s. Now, the slope between points $(k, 1250)$ and $(108, 500)$ must be the same as the slope between $(108, 500)$ and $(312, 1500)$.

$$\begin{aligned} \therefore \frac{1250-500}{k-108} &= \frac{1500-500}{312-108} \\ \frac{750}{k-108} &= \frac{1000}{204} \\ (k-108)1000 &= 750(204) \quad \longleftarrow \text{Cross multiply.} \\ 1000k - 108\,000 &= 153\,000 \\ 1000k &= 261\,000 \\ k &= \frac{261\,000}{1000} \\ &= 261 \end{aligned}$$

b. k is expressed in seconds $(k = 261 \text{ s})$.

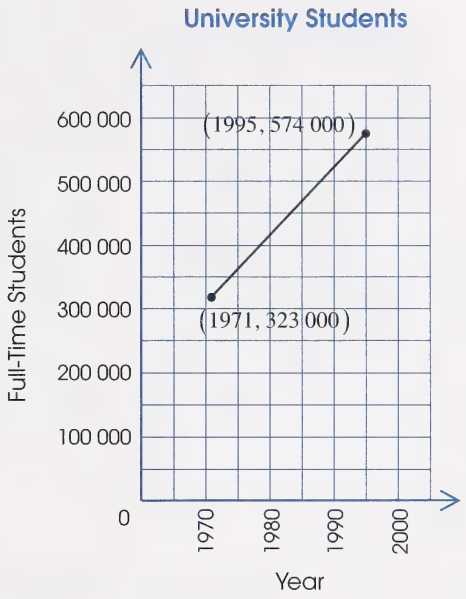
c. k represents the time it took Marnie McBean and Kathleen Heddle to row 1250 m.

5. Textbook questions 1, 5, and 6 of "Applications and Problem Solving," pp. 271 and 272

1. The average yearly rate of change is the slope of the graph.

$$m = \frac{574\,000 - 323\,000}{1995 - 1971}$$
$$= \frac{251\,000}{24}$$
$$\doteq 10\,458$$

The yearly rate of change, to the nearest 100 students, is about 10 500 students/year.



5. a. The rate of change is the slope of the graph.

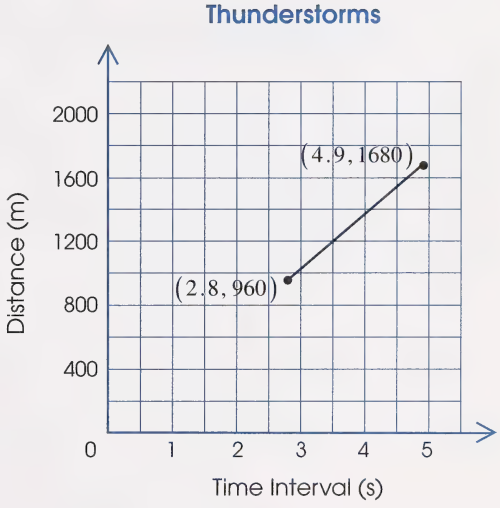
$$m = \frac{1680 - 960}{4.9 - 2.8}$$
$$= \frac{720}{2.1}$$
$$\doteq 343$$

The rate of change is about 343 m/s.

b. The difference between the speed of light and the speed of sound in air is approximately 343 m/s.

c. distance = rate × time
$$\doteq 343 \times 3.7$$
$$\doteq 1269$$

The distance from the storm, to the nearest 10 m, is approximately 1270 m.



Section 1: Activity 2 (continued)

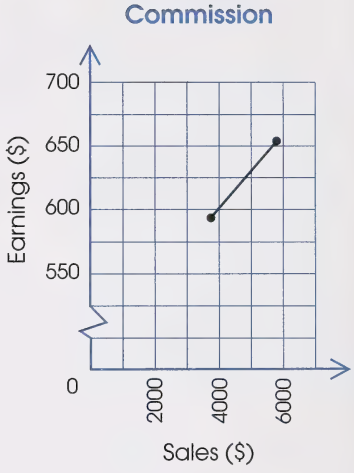
d. $\text{time} = \dfrac{\text{distance}}{\text{rate}}$

$$\doteq \frac{2500}{343}$$

$$\doteq 7.3 \text{ s}$$

The time interval, to the nearest tenth of a second, is about 7.3 s.

6. a. The rate of commission is the slope of the graph.



$$\therefore m = \frac{652.50 - 594}{5750 - 3800}$$

$$= \frac{58.50}{1950}$$

$$= 0.03$$

Mariko's rate of commission is 0.03 or 3%.

b. Mariko earned \$594 when her sales were \$3800. Let S represent Mariko's base salary.

$$\text{Earnings} = \text{base salary} + \text{commission}$$

$$594 = S + 3\% \text{ of } 3800$$

$$594 = S + 0.03(3800)$$

$$594 = S + 114$$

$$S + 114 = 594$$

$$S = 480$$

Mariko's weekly base salary is \$480.

c. $\text{Earnings} = \text{base salary} + \text{commission}$

$$= 480 + 3\% \text{ of } 4325$$

$$= 480 + 0.03(4325)$$

$$= 480 + 129.75$$

$$= 609.75$$

Mariko would earn \$609.75.

6. Textbook question "Logic Power," p. 269

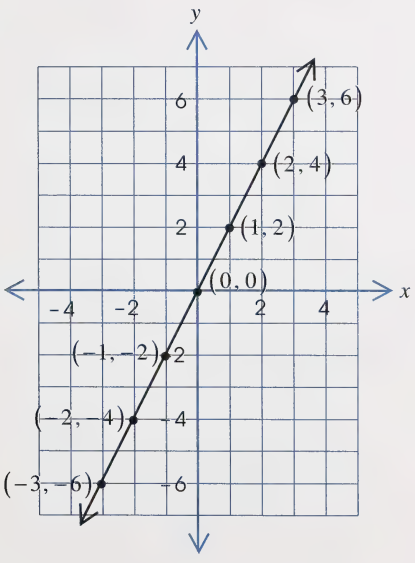
Each box represents a pail. The number inside each box represents the amount of water in the pail. Each row of boxes represents the contents of the pails after each pouring. Answers may vary. For example,

7-L Pail	3-L Pail	10-L Pail	
7 L	0 L	3 L	← Fill the 7-L pail from the 10-L pail.
4 L	3 L	3 L	← Fill the 3-L pail from the 7-L pail.
4 L	0 L	6 L	← Pour the contents of the 3-L pail into the 10-L pail.
1 L	3 L	6 L	← Fill the 3-L pail from the 7-L pail.
1 L	0 L	9 L	← Pour the contents of the 3-L pail into the 10-L pail.
0 L	1 L	9 L	← Pour the contents of the 7-L pail into the 3-L pail.
7 L	1 L	2 L	← Fill the 7-L pail from the 10-L pail.
5 L	3 L	2 L	← Fill the 3-L pail from the 7-L pail.
5 L	0 L	5 L	← Pour the contents of the 3-L pail into the 10-L pail.

Thus, 9 pourings would be required to end up with 5 L in the 7-L pail and 5 L in the 10-L pail.

Section 1: Activity 3

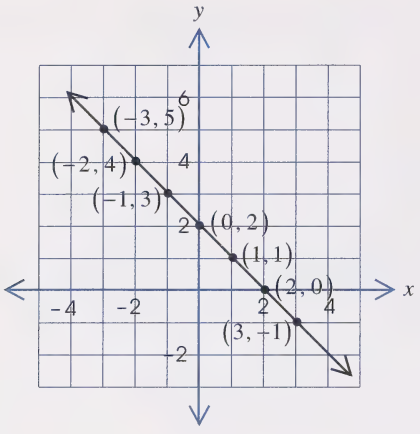
1. a.



The equation of the line is $y = 2x$ because the y-coordinate is double the x-coordinate for each ordered pair.

Section 1: Activity 3 (continued)

b.



The sum of the coordinates for each ordered pair is 2. Therefore, the equation of the line is as follows:

$$x + y = 2$$

This equation may be rewritten with y isolated on the left side.

$$\begin{aligned} x + y &= 2 \\ x + y - x &= 2 - x \\ y &= -x + 2 \end{aligned}$$

2. a. Textbook questions a. and b. of "Explore: Develop a Method," p. 278

a. Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (x, y)$.

$$\begin{aligned} \therefore m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{y - 0}{x - 0} \\ &= \frac{y}{x} \end{aligned}$$

b. Because the slope is 2.5,

$$\begin{aligned} 2.5 &= \frac{y}{x} \\ \frac{y}{x} &= 2.5 \end{aligned}$$

b. Textbook questions 1 to 4 of "Inquire," p. 278

1. a.

$$\begin{aligned} \frac{y}{x} &= 2.5 \\ \frac{y}{x}(x) &= 2.5(x) \\ y &= 2.5x \end{aligned}$$

b. 2.5 is the slope.

2. a. When $x = 20$, $y = 2.5(20)$

$= 50$

The circumference of the tree is 50 cm.

b. When $x = 85$, $y = 2.5(85)$

$= 212.5$

The circumference of the tree is 212.5 cm.

3. a. When $y = 80$, $y = 2.5x$

$$80 = 2.5x$$
$$2.5x = 80$$
$$\frac{2.5x}{2.5} = \frac{80}{2.5}$$
$$x = 32$$

The tree is 32 years old.

b. When $y = 175$, $y = 2.5x$

$$175 = 2.5x$$
$$2.5x = 175$$
$$\frac{2.5x}{2.5} = \frac{175}{2.5}$$
$$x = 70$$

The tree is 70 years old.

4.
$$y = 2.5x$$
$$2.5x = y$$
$$2.5x - y = y - y$$
$$2.5x - y = 0$$

3. a.
$$y = \frac{2}{3}x + 4$$
$$\frac{2}{3}x + 4 = y \quad \text{← Interchange the sides to obtain } x \text{ on the left side.}$$
$$3\left(\frac{2}{3}x + 4\right) = 3(y) \quad \text{← Multiply by 3 to clear the fractions.}$$
$$2x + 12 = 3y$$
$$2x + 12 - 3y = 3y - 3y$$
$$2x - 3y + 12 = 0$$

b.
$$y - 2 = \frac{3}{5}x - 7$$
$$5(y - 2) = 5\left(\frac{3}{5}x - 7\right) \quad \text{← Multiply both sides by 5 to clear the fraction.}$$
$$5y - 10 = 3x - 35$$
$$3x - 35 = 5y - 10 \quad \text{← Interchange the sides to obtain } x \text{ on the left side.}$$
$$3x - 35 - 5y = 5y - 10 - 5y$$
$$3x - 5y - 35 = -10$$
$$3x - 5y - 35 + 10 = -10 + 10$$
$$3x - 5y - 25 = 0$$

Section 1: Activity 3 (continued)

4. Textbook questions 1, 3, 11, 13, 15, and 17 of "Practice," p. 282

1. Use the point-slope form, $y-y_1=m(x-x_1)$.
Let $(x_1, y_1)=(2,3)$ and $m=4$.

$$\begin{aligned}
\therefore\ y-y_1 &= m(x-x_1)\\
y-3 &= 4(x-2)\\
y-3 &= 4x-8\\
4x-8 &= y-3\\
4x-8-y &= y-3-y\\
4x-y-8 &= -3\\
4x-y-8+3 &= -3+3\\
4x-y-5 &= 0
\end{aligned}$$

3. Let $(x_1, y_1)=(-5,2)$ and $m=2$.

$$\begin{aligned}
\therefore\ y-y_1 &= m(x-x_1)\\
y-2 &= 2[x-(-5)]\\
y-2 &= 2(x+5)\\
y-2 &= 2x+10\\
2x+10 &= y-2\\
2x+10-y &= y-2-y\\
2x-y+10 &= -2\\
2x-y+10+2 &= -2+2\\
2x-y+12 &= 0
\end{aligned}$$

11. Let $(x_1, y_1)=(-3,4)$ and $m=-\frac{1}{3}$.

$$\begin{aligned}
\therefore\ y-y_1 &= m(x-x_1)\\
y-4 &= -\frac{1}{3}[x-(-3)]\\
y-4 &= -\frac{1}{3}(x+3)\\
y-4 &= -\frac{1}{3}x-1\\
3(y-4) &= 3\left(-\frac{1}{3}x-1\right)\\
3y-12 &= -x-3\\
3y-12+x &= -x-3+x\\
x+3y-12 &= -3\\
x+3y-12+3 &= -3+3\\
x+3y-9 &= 0
\end{aligned}$$

13. Let $(x_1, y_1)=\left(\frac{1}{2}, 6\right)$ and $m=\frac{4}{3}$.

$$\begin{aligned}
\therefore\ y-y_1 &= m(x-x_1)\\
y-6 &= \frac{4}{3}\left(x-\frac{1}{2}\right)\\
y-6 &= \frac{4}{3}x-\frac{\overset{2}{\cancel{4}}}{3}\left(\frac{1}{\cancel{2}_{1}}\right)\\
y-6 &= \frac{4}{3}x-\frac{2}{3}\\
3(y-6) &= 3\left(\frac{4}{3}x-\frac{2}{3}\right)\\
3y-18 &= 4x-2\\
4x-2 &= 3y-18\\
4x-2-3y &= 3y-18-3y\\
4x-3y-2 &= -18\\
4x-3y-2+18 &= -18+18\\
4x-3y+16 &= 0
\end{aligned}$$

15. Let $(x_1, y_1) = (1, 5)$ and $m = 1$.

$$\begin{aligned}
\therefore\ y - y_1 &= m(x - x_1) \\
y - 5 &= 1(x - 1) \\
y - 5 &= x - 1 \\
x - 1 &= y - 5 \\
x - 1 - y &= y - 5 - y \\
x - y - 1 &= -5 \\
x - y - 1 + 5 &= -5 + 5 \\
x - y + 4 &= 0
\end{aligned}$$

To find two other points on the line, it is convenient to solve the equation for y.

$$\begin{aligned}
x - y + 4 &= 0 \\
x - y + 4 + y &= 0 + y \\
x + 4 &= y \\
y &= x + 4
\end{aligned}$$

When $x = 0$, $y = 0 + 4$
$= 4$

When $x = 2$, $y = 2 + 4$
$= 6$

Two other points on the line are $(0, 4)$ and $(2, 6)$.

Check

Substitute the coordinates of the points into $x - y + 4 = 0$.

For $(0, 4)$,

LS	RS
$x - y + 4$	0
$= 0 - 4 + 4$	
$= 0$	

LS = RS

For $(2, 6)$,

LS	RS
$x - y + 4$	0
$= 2 - 6 + 4$	
$= 0$	

LS = RS

Both points satisfy the original equation, $x - y + 4 = 0$.

Section 1: Activity 3 (continued)

17. Let $(x_1, y_1) = (4, 3)$ and $m = -1$.

$$\begin{aligned}
\therefore\ y - y_1 &= m(x - x_1) \\
y - 3 &= -1(x - 4) \\
y - 3 &= -x + 4 \\
y - 3 + x &= -x + 4 + x \\
x + y - 3 &= 4 \\
x + y - 3 - 4 &= 4 - 4 \\
x + y - 7 &= 0
\end{aligned}$$

To find two other points on the line, it is convenient to solve the equation for y.

$$\begin{aligned}
x + y - 7 &= 0 \\
x + y - 7 - x &= 0 - x \\
y - 7 &= -x \\
y - 7 + 7 &= -x + 7 \\
y &= -x + 7
\end{aligned}$$

When $x = 0$, $y = -(0) + 7$
$= 7$

When $x = 1$, $y = -(1) + 7$
$= 6$

Two other points on the line are $(0, 7)$ and $(1, 6)$.

Check

Substitute the coordinates of the points into $x + y - 7 = 0$.

For $(0, 7)$,

LS	RS
$x + y - 7$	0
$= 0 + 7 - 7$	
$= 0$	

LS = RS

For $(1, 6)$,

LS	RS
$x + y - 7$	0
$= 1 + 6 - 7$	
$= 0$	

LS = RS

Both points satisfy the original equation, $x + y - 7 = 0$.

5. Textbook questions 21, 25, 28, and 38 of "Practice," p. 282

21. Use $(3,4)$ and $(4,6)$ to find the slope of the line. Let $(x_1, y_1) = (3,4)$ and $(x_2, y_2) = (4,6)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6-4}{4-3} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

Use the point-slope form and either of the two given points. Let $(x_1, y_1) = (3,4)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 2(x-3) \\ y - 4 &= 2x - 6 \\ 2x - 6 &= y - 4 \\ 2x - 6 - y &= y - 4 - y \\ 2x - y - 6 &= -4 \\ 2x - y - 6 + 4 &= -4 + 4 \\ 2x - y - 2 &= 0 \end{aligned}$$

Check

Because $(3,4)$ was used to find the equation, you can use $(4,6)$ to check the answer.

LS	RS
$2x - y - 2$	0
$= 2(4) - 6 - 2$	
$= 8 - 6 - 2$	
$= 0$	

LS = RS

Section 1: Activity 3 (continued)

25. Use $(-1,-2)$ and $(3,0)$ to find the slope of the line. Let $(x_1, y_1)=(-1,-2)$ and $(x_2, y_2)=(3,0)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0-(-2)}{3-(-1)} \\ &= \frac{0+2}{3+1} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

Use the point-slope form and either of the two given points. Let $(x_1, y_1)=(-1,-2)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y-(-2) &= \frac{1}{2}[x-(-1)] \\ y+2 &= \frac{1}{2}(x+1) \\ y+2 &= \frac{1}{2}x+\frac{1}{2} \\ 2(y+2) &= 2\left(\frac{1}{2}x+\frac{1}{2}\right) \\ 2y+4 &= x+1 \\ x+1 &= 2y+4 \\ x+1-2y &= 2y+4-2y \\ x-2y+1 &= 4 \\ x-2y+1-4 &= 4-4 \\ x-2y-3 &= 0 \end{aligned}$$

Check

Use $(3, 0)$ to check the answer.

LS	RS
$x-2y-3$	0
$=3-2(0)-3$	
$=3-0-3$	
$=0$	

LS = RS

28. Use $(8, -7)$ and $(-6, -7)$ to find the slope of the line. Let $(x_1, y_1) = (8, -7)$ and $(x_2, y_2) = (-6, -7)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-7-(-7)}{-6-8} \\ &= \frac{-7+7}{-14} \\ &= \frac{0}{-14} \\ &= 0 \end{aligned}$$

Use the point-slope form. Let $(x_1, y_1) = (8, -7)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y-(-7) &= 0(x-8) \\ y+7 &= 0 \end{aligned}$$

Check

Use $(-6, -7)$ to check the answer.

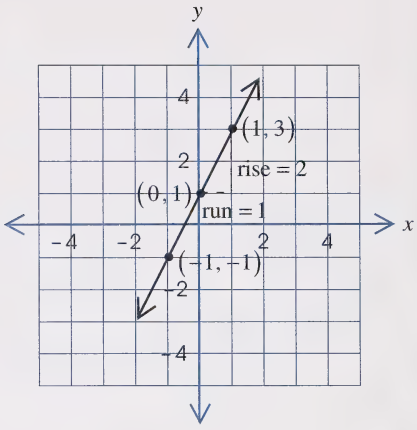
LS	RS
$y+7$	0
$=-7+7$	
$=0$	

LS = RS

Section 1: Activity 3 (continued)

38. $\text{slope} = \dfrac{\text{rise}}{\text{run}}$

$$m = \frac{2}{1}$$
$$= 2$$



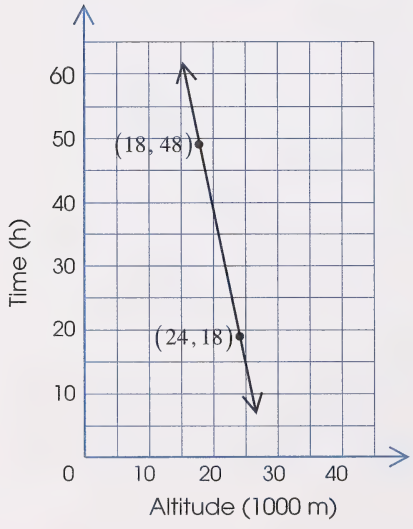
Use any point through which the line passes in the point-slope form. Let $(x_1, y_1) = (1, 3)$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 2(x - 1)$$
$$y - 3 = 2x - 2$$
$$2x - 2 = y - 3$$
$$2x - 2 - y = y - 3 - y$$
$$2x - y - 2 = -3$$
$$2x - y - 2 + 3 = -3 + 3$$
$$2x - y + 1 = 0$$

6. Textbook questions 41 and 46 of "Applications and Problem Solving," p. 283

41. a.

Aviation



b. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{48-18}{18-24}$

$= \dfrac{30}{-6}$

$= -5$

Use the point-slope form. Since altitude = a and time = t, the point-slope form can be revised as $t - t_1 = m(a - a_1)$. Let $(a_1, t_1) = (24, 18)$.

$$\begin{aligned} t - 18 &= -5(a - 24) \\ t - 18 &= -5a + 120 \\ t - 18 + 5a &= -5a + 120 + 5a \\ 5a + t - 18 &= 120 \\ 5a + t - 18 - 120 &= 120 - 120 \\ 5a + t - 138 &= 0 \end{aligned}$$

c. It would be more convenient to solve this equation for t.

$$\begin{aligned} 5a + t - 138 &= 0 \\ t &= -5a + 138 \end{aligned}$$

To find the time for which the plane can fly at 20 000 m, substitute 20 for a because the altitude is in thousands of metres.

$t = -5(20) + 138$

$= -100 + 138$

$= 38$

The plane can fly for 38 h at an altitude of 20 000 m.

For an altitude of 26 000 m, substitute 26 for a.

$t = -5(26) + 138$

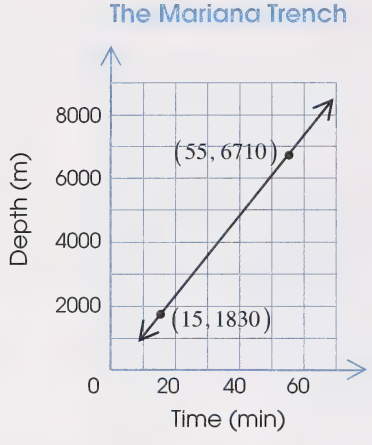
$= -130 + 138$

$= 8$

The plane can fly for 8 h at 26 000 m.

Section 1: Activity 3 (continued)

46. a.



$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6710 - 1830}{55 - 15} \\ &= \frac{4880}{40} \\ &= 122 \end{aligned}$$

b. The slope represents a rate of change of 122 m/min.

c. Use the point-slope form. Since time $= t$ and depth $= d$, the point-slope form can be revised as $d - d_1 = m(t - t_1)$. Let $(t_1, d_1) = (15, 1830)$.

$$\begin{aligned} d - 1830 &= 122(t - 15) \\ d - 1830 &= 122t - 1830 \\ 122t - 1830 &= d - 1830 \\ 122t - 1830 - d &= d - 1830 - d \\ 122t - d - 1830 &= -1830 \\ 122t - d - 1830 + 1830 &= -1830 + 1830 \\ 122t - d &= 0 \end{aligned}$$

d. First, solve for t.

$$\begin{aligned} 122t - d &= 0 \\ 122t &= d \\ t &= \frac{d}{122} \end{aligned}$$

If $d = 11\,000$ m, then $t = \frac{11\,000}{122}$

$\doteq 90$

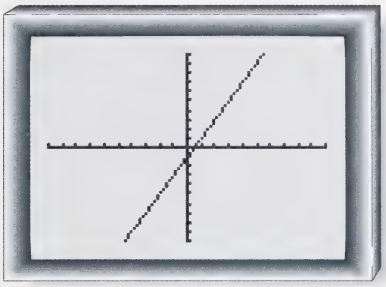
It will take Deep Flight approximately 90 min to reach the bottom of the Mariana Trench.

7. a. Textbook questions 1 and 2 of Investigation 1, "Slope and *y*-Intercept," p. 284

1. a. Enter the following keystrokes:

[Y=] [2] [X,T,Θ,n] [−] [1] [GRAPH]

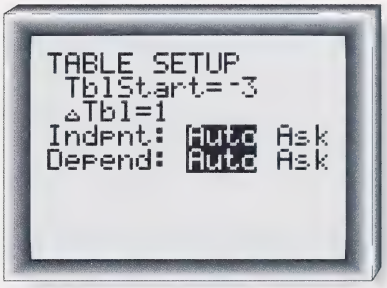
Your graph should look like the one on the right.



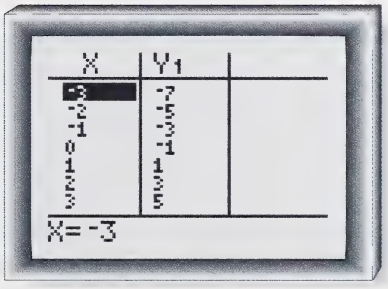
b. To use the table feature, you may wish to first set the table. Enter the following keystrokes:

[2nd] [TBLSET]

Your calculator display should look like the one on the right.



If your display differs, use the arrow keys to move the cursor on the screen to change your display. Remember to press [ENTER] each time you make a change. These settings will give you a table of values beginning with an x-coordinate of -3. With these settings, when you graph $y = 2x - 1$ and then press [2nd] [TABLE], your calculator should show the following:



The y-intercept is -1, obtained from the ordered pair $(0, -1)$ in the table.

c. A second ordered pair from the table is $(1, 1)$.

Section 1: Activity 3 (continued)

d. Let $(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (1, 1)$.

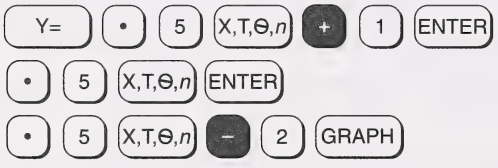
$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{1-(-1)}{1-0}$$
$$= \frac{1+1}{1}$$
$$= 2$$

e. In $y = 2x - 1$, the coefficient of x is the slope and the constant term is the y-intercept.

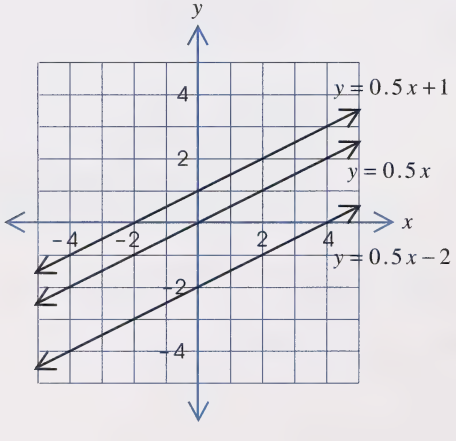
2. a. The slope is 2, and the y-intercept is 1.
b. The slope is 0.5, and the y-intercept is 2.
c. The slope is -1, and the y-intercept is -1.

b. Textbook questions 1 to 5 of Investigation 2, "A Family of Lines," p. 284

1. To graph all three equations so that they appear in the same viewing window, enter the following keystrokes:



The display on your calculator will be similar to the following graph:



2.

Equation	*y*-Intercept
$y = 0.5x + 1$	1
$y = 0.5x$	0
$y = 0.5x - 2$	−2

3.

Equation	Slope
$y = 0.5x + 1$	0.5
$y = 0.5x$	0.5
$y = 0.5x - 2$	0.5

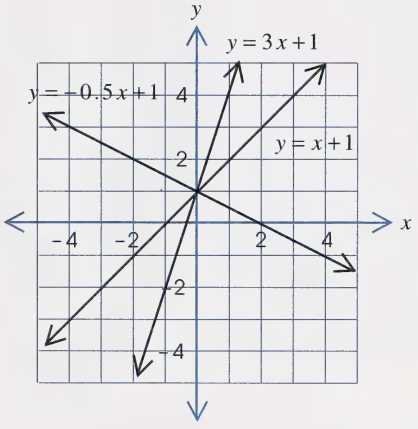
4. The lines have the same slopes, making them parallel, but they have different *y*-intercepts.

5. Answers will vary. A sample answer is given.

$y = 0.5x - 3$

c. Textbook questions 1 to 5 of Investigation 3, "A Different Family of Lines," p. 285

1. The display on your calculator should be similar to the following graph:



2. The *y*-intercept of each line is 1.

Section 1: Activity 3 (continued)

3.

Equation	Slope
$y=3x+1$	3
$y=x+1$	1
$y=-0.5x+1$	-0.5

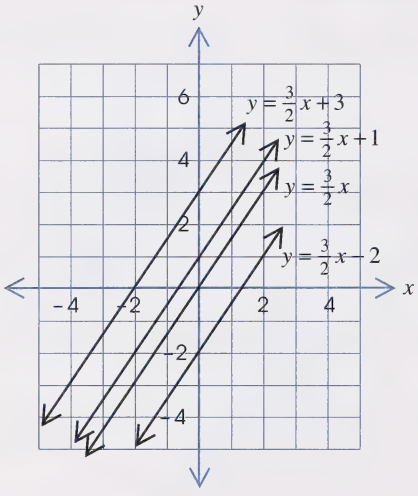
4. These lines have different slopes but the same y-intercept. Their graphs all pass through the point $(0,1)$.

5. Answers will vary. A sample answer is given.

$$y=2x+1$$

d. Textbook questions 1 to 4 of Investigation 4, "Related Lines," p. 285

1. The display on your calculator should be similar to the following graph:



2.

Equation	y-Intercept
$y=\frac{3}{2}x+3$	3
$y=\frac{3}{2}x+1$	1
$y=\frac{3}{2}x$	0
$y=\frac{3}{2}x-2$	-2

3. The slope of each line is $\frac{3}{2}$.

4. The four lines belong to a family of parallel lines, each with the same slope but different y-intercepts.

8. a. Textbook questions a. to e. of "Explore: Use the Graphs," p. 286

	Equation	$y =$ Form	Points	Slope	y-Intercept
a.	$y = 2x + 3$	$y = 2x + 3$	$(0,3),\ (1,5)$	2	3
b.	$x + y = 2$	$y = -x + 2$	$(0,2),\ (1,1)$	-1	2
c.	$y - x = 0$	$y = x$	$(0,0),\ (1,1)$	1	0
d.	$3x + y + 1 = 0$	$y = -3x - 1$	$(0,-1),\ (1,-4)$	-3	-1
e.	$2x + 2y + 6 = 0$	$y = -x - 3$	$(0,-3),\ (1,-4)$	-1	-3

b. Textbook questions 1 to 4 of "Inquire," p. 287

1. a. When the equation is solved for y, the slope is equal to the numerical coefficient of x.

b. When the equation is solved for y, the y-intercept is equal to the constant term.

2. a.
$$3y = 6x + 1$$
$$\frac{3y}{3} = \frac{6x + 1}{3}$$
$$y = 2x + \frac{1}{3}$$

The slope is 2, and the y-intercept is $\frac{1}{3}$.

b.
$$2x + y + 4 = 0$$
$$2x + y + 4 - 2x = 0 - 2x$$
$$y + 4 = -2x$$
$$y + 4 - 4 = -2x - 4$$
$$y = -2x - 4$$

The slope is -2, and the y-intercept is -4.

Section 1: Activity 3 (continued)

c.
$$\begin{aligned} x-y&=2\\ x-y-x&=2-x\\ -y&=-x+2\\ -1(-y)&=-1(-x+2)\\ y&=x-2 \end{aligned}$$

The slope is 1, and the y-intercept is -2.

d.
$$\begin{aligned} x+2y-1&=0\\ x+2y-1-x&=0-x\\ 2y-1&=-x\\ 2y-1+1&=-x+1\\ 2y&=-x+1\\ \frac{2y}{2}&=\frac{-x+1}{2}\\ y&=-\frac{1}{2}x+\frac{1}{2} \end{aligned}$$

The slope is $-\frac{1}{2}$, and the y-intercept is $\frac{1}{2}$.

3. The equation $y=mx+b$ is called the slope and y-intercept form of a linear equation because m represents the slope and b represents the y-intercept of a line.

4. a.
$$\begin{aligned} 150t+a&=3000\\ 150t+a-a&=3000-a\\ \frac{150t}{150}&=\frac{-a+3000}{150}\\ t&=-\frac{1}{150}a+20 \end{aligned}$$

b. If $a=0$, then $t=-\frac{1}{150}(0)+20$
$$=20$$

The ground-level temperature is 20°C.

c. The slope of the graph is $-\frac{1}{150}$°C/m. This rate indicates that the outside air temperature drops 1°C for every 150 m increase in altitude.

9. a. Textbook questions 1 and 3 of "Practice," p. 288

1. $y = 3x + 1$

$y = mx + b$

The slope, m, is 3, and the y-intercept, b, is 1.

3. $y = -4x + 3$

$y = mx + b$

The slope, m, is -4, and the y-intercept, b, is 3.

b. Textbook questions 9, 11, and 13 of "Practice," p. 288

9.
$$4x + 2y = 3$$
$$4x + 2y - 4x = 3 - 4x$$
$$2y = -4x + 3$$
$$\frac{2y}{2} = \frac{-4x + 3}{2}$$
$$y = -2x + \frac{3}{2}$$
$$y = mx + b$$

The slope, m, is -2, and the y-intercept, b, is $+\frac{3}{2}$.

11.
$$3x + 2y + 6 = 0$$
$$3x + 2y + 6 - 3x = 0 - 3x$$
$$2y + 6 = -3x$$
$$2y + 6 - 6 = -3x - 6$$
$$2y = -3x - 6$$
$$\frac{2y}{2} = \frac{-3x - 6}{2}$$
$$y = -\frac{3}{2}x - 3$$
$$y = mx + b$$

The slope, m, is $-\frac{3}{2}$, and the y-intercept, b, is -3.

13.
$$x - 3y - 9 = 0$$
$$x - 3y - 9 + 3y = 0 + 3y$$
$$x - 9 = 3y$$
$$3y = x - 9$$
$$\frac{3y}{3} = \frac{x - 9}{3}$$
$$y = \frac{1}{3}x - 3$$
$$y = mx + b$$

The slope, m, is $\frac{1}{3}$, and the y-intercept, b, is -3.

Section 1: Activity 3 (continued)

c. Textbook questions 19, 23, and 26 of "Practice," p. 288

19. Because $m = 2$ and $b = 3$,

$$\begin{aligned} y &= mx + b \\ y &= 2x + 3 && \leftarrow \text{slope and } y\text{-intercept form} \\ y - y &= 2x + 3 - y \\ 0 &= 2x - y + 3 \\ 2x - y + 3 &= 0 && \leftarrow \text{standard form} \end{aligned}$$

23. Because $m = \frac{1}{2}$ and $b = 1$,

$$\begin{aligned} y &= mx + b \\ y &= \frac{1}{2}x + 1 && \leftarrow \text{slope and } y\text{-intercept form} \\ 2(y) &= 2\left(\frac{1}{2}x + 1\right) \\ 2y &= x + 2 \\ 2y - 2y &= x + 2 - 2y \\ 0 &= x - 2y + 2 \\ x - 2y + 2 &= 0 && \leftarrow \text{standard form} \end{aligned}$$

26. Because $m = -\frac{2}{5}$ and $b = -\frac{1}{3}$,

$$\begin{aligned} y &= mx + b \\ y &= -\frac{2}{5}x - \frac{1}{3} && \leftarrow \text{slope and } y\text{-intercept form} \\ 15(y) &= 15\left(-\frac{2}{5}x - \frac{1}{3}\right) \\ 15y &= -6x - 5 \\ 15y + 6x &= -6x - 5 + 6x \\ 6x + 15y &= -5 \\ 6x + 15y + 5 &= -5 + 5 \\ 6x + 15y + 5 &= 0 && \leftarrow \text{standard form} \end{aligned}$$

d. Textbook questions 27 and 29 of "Practice," p. 288

27. $y = 2x + 5$

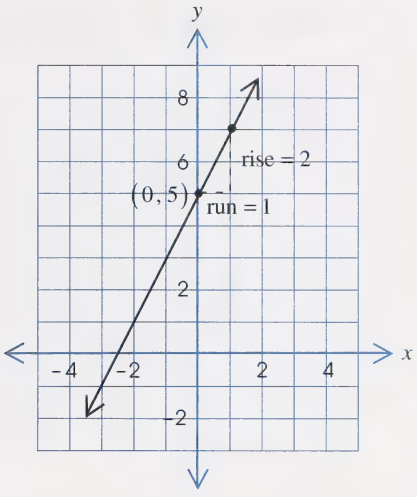
$y = mx + b$

$\therefore\ m = 2$ and $b = 5$

First, plot the point $(0, 5)$.

Because $m = 2 = \frac{2}{1}$, count 1 unit to the right and 2 units up from $(0, 5)$.

Finally, draw a line through the two points.



29. $y = -x + 4$

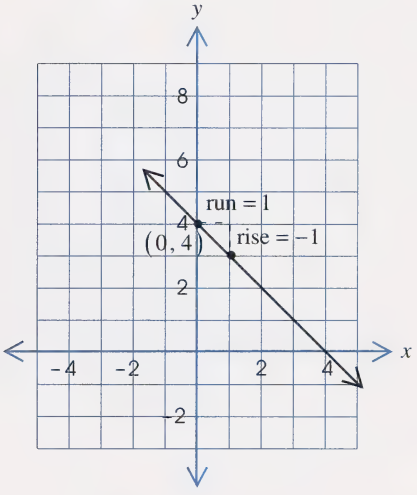
$y = mx + b$

$\therefore\ m = -1$ and $b = 4$

First, plot the point $(0, 4)$.

Because $m = -1 = \frac{-1}{1}$, count 1 unit to the right and 1 unit down from $(0, 4)$.

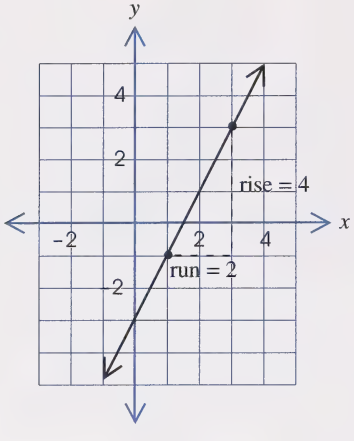
Finally, draw a line through the two points.



Section 1: Activity 3 (continued)

e. Textbook questions 34 and 36 of "Practice," p. 288

34.

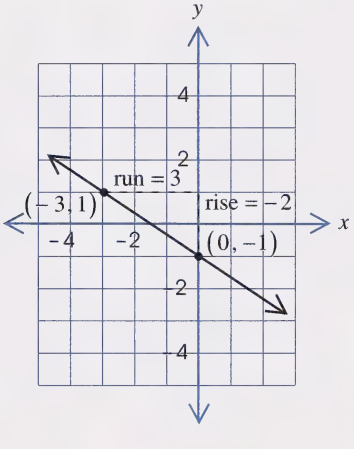


$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{2}$$
$$= 2$$

Because $m = 2$ and $b = -3$, the equation of the line is as follows:

$$y = mx + b$$
$$y = 2x - 3$$

36.



$$m = \frac{\text{rise}}{\text{run}}$$
$$= -\frac{2}{3}$$

Because $m = -\frac{2}{3}$ and $b = -1$, the equation of the line is as follows:

$$y = mx + b$$
$$y = -\frac{2}{3}x - 1$$

f. Textbook questions 37.a., 38.a., 41, and 44 of "Applications and Problem Solving," pp. 288 and 289

37. a. The line $y = 2x + b$ passes through point $(4, 2)$. Therefore, the coordinates of point $(4, 2)$ must satisfy the equation.

If $x = 4$ and $y = 2$, then

$$\begin{aligned} 2 &= 2(4) + b \\ 2 &= 8 + b \\ b + 8 &= 2 \\ b + 8 - 8 &= 2 - 8 \\ b &= -6 \end{aligned}$$

38. a. The line $y = mx + 3$ passes through $(2, 1)$. Therefore, the coordinates of point $(2, 1)$ must satisfy the equation.

If $x = 2$ and $y = 1$, then

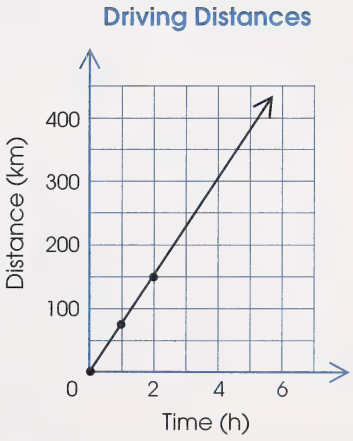
$$\begin{aligned} 1 &= m(2) + 3 \\ 1 &= 2m + 3 \\ 2m + 3 &= 1 \\ 2m + 3 - 3 &= 1 - 3 \\ 2m &= -2 \\ \frac{2m}{2} &= \frac{-2}{2} \\ m &= -1 \end{aligned}$$

41. a. If the average distance travelled every hour is 75 km, the following table may be used to draw the graph of distance versus time.

Time, t (h)	0	1	2
Distance, d (km)	0	75	150

$\therefore\ d = 75t$

slope



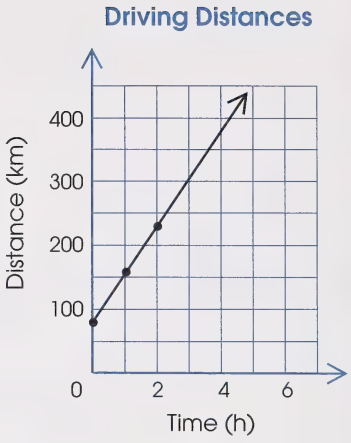
Section 1: Activity 3 (continued)

b. Because Virden is 80 km west of Brandon, 80 km must be added to the distances given in question 41.a. The following table may be used to draw the graph of distance versus time.

Time, t (h)	0	1	2
Distance, d (km)	80	155	230

$\therefore\ d = 75t + 80$

slope, vertical intercept



c. Use the equation to find the distance from Virden.

$$d = 75t$$
$$= 75(7)$$
$$= 525 \text{ km}$$

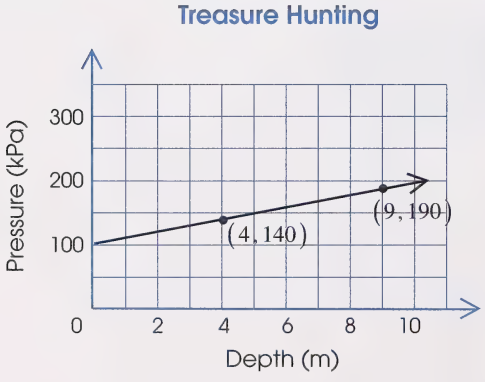
The distance from Virden to Swift Current is 525 km.

Use the equation to find the distance from Brandon.

$$d = 75t + 80$$
$$= 75(7) + 80$$
$$= 605 \text{ km}$$

The distance from Brandon to Swift Current is 605 km.

44. a.



b. First, find the slope of the line using the two points given. Let $(d_1, p_1) = (4, 140)$ and $(d_2, p_2) = (9, 190)$.

$$\begin{aligned} m &= \frac{p_2 - p_1}{d_2 - d_1} \\ &= \frac{190 - 140}{9 - 4} \\ &= \frac{50}{5} \\ &= 10 \text{ kPa / m} \end{aligned}$$

Now use the point-slope to determine the equation of the line. Let $(d_1, p_1) = (4, 140)$.

$$\begin{aligned} p - p_1 &= m(d - d_1) \\ p - 140 &= 10(d - 4) \\ p - 140 &= 10d - 40 \\ p - 140 + 140 &= 10d - 40 + 140 \\ p &= 10d + 100 \end{aligned}$$

c. The slope represents the change in pressure (kPa) for every additional metre of depth.

$$\text{slope} = 10 \text{ kPa/m}$$

d. The p-intercept is 100 kPa; it represents the pressure at the surface of the water.

e. If $p = 200$, then

$$\begin{aligned} 200 &= 10d + 100 \\ 10d + 100 &= 200 \\ 10d + 100 - 100 &= 200 - 100 \\ 10d &= 100 \\ \frac{10d}{10} &= \frac{100}{10} \\ d &= 10 \end{aligned}$$

At a depth of 10 m, the pressure is double the pressure at the surface.

Section 1: Activity 3 (continued)

f. If $d = 6$, then $p = 10d + 100$

$= 10(6) + 100$

$= 60 + 100$

$= 160$

The pressure on the divers was 160 kPa.

10. a. 52° north latitude, 106° west longitude
b. 43° north latitude, 82° west longitude
c. Answers will vary.

Section 1: Activity 4

1. a. Textbook questions a. to c. of "Explore: Use the Graphs—Parallel Lines," p. 291

a. and b. In the diagram, line AB passes through $A(0, 4)$ and $B(4, 2)$.

In the diagram, line CD passes through $C(-5, 0)$ and $D(-1, -2)$.

c. $m_{AB} = \dfrac{2-4}{4-0}$

$= \dfrac{-2}{4}$

$= -\dfrac{1}{2}$

$m_{CD} = \dfrac{-2-0}{-1-(-5)}$

$= \dfrac{-2}{-1+5}$

$= \dfrac{-2}{4}$

$= -\dfrac{1}{2}$

The slope of line AB is the same as the slope of line CD.

b. Textbook questions a. to c. of "Explore: Use the Graphs—Perpendicular Lines," p. 291

a. In the diagram, point P is $(2, 3)$.

b. The coordinates of A are $(5, 2)$, and the coordinates of B are $(1, 0)$.

c. $m_{PA} = \frac{2-3}{5-2}$ $\qquad$ $m_{PB} = \frac{0-3}{1-2}$

$= \frac{-1}{3}$ $\qquad$ $= \frac{-3}{-1}$

$= -\frac{1}{3}$ $\qquad$ $= 3$

The slope of line *PA* is the negative reciprocal of the slope of line *PB*.

c. Textbook questions 1 to 6 of "Inquire," p. 292

1. The slopes of parallel lines are equal.

2. The slopes of perpendicular, oblique lines are negative reciprocals of each other.

3. The product of the slopes of two perpendicular lines is -1. For example, if the slopes of two perpendicular lines are 3 and $-\frac{1}{3}$, then $3\left(-\frac{1}{3}\right) = -1$.

4. a.
$$2x - y = 2$$
$$2x - y - 2x = 2 - 2x$$
$$-y = -2x + 2$$
$$-1(-y) = -1(-2x + 2)$$
$$y = 2x - 2$$
$$y = mx + b$$

The slope is 2.

b.
$$x + y = -2$$
$$x + y - x = -2 - x$$
$$y = -x - 2$$
$$y = mx + b$$

The slope is -1.

c.
$$x - y = 1$$
$$x - y - x = 1 - x$$
$$-y = -x + 1$$
$$-1(-y) = -1(-x + 1)$$
$$y = x - 1$$
$$y = mx + b$$

The slope is 1.

d.
$$y - 2x = -3$$
$$y - 2x + 2x = -3 + 2x$$
$$y = 2x - 3$$
$$y = mx + b$$

The slope is 2.

Section 1: Activity 4 (continued)

5. $2x - y = 2$ and $y - 2x = -3$ are equations of parallel lines because each line has a slope of 2.

6. $x + y = -2$ and $x - y = 1$ are equations of perpendicular lines because the product of their slopes is -1.

$$\begin{aligned}(\text{slope of } x + y = -2)(\text{slope of } x - y = 1) &= -1(1) \\ &= -1\end{aligned}$$

2. a. Textbook questions 1 to 3 of "Practice," p. 294

1. $m_1 \neq m_2$ $\qquad m_1 \times m_2 = \frac{2}{3}\left(\frac{3}{2}\right)$
$= 1$

Therefore, the lines are neither parallel nor perpendicular.

2. Because $m_1 = \frac{1}{2}$ and $m_2 = \frac{4}{8} = \frac{1}{2}$, the lines are parallel.

3. $m_1 \times m_2 = -3\left(\frac{1}{3}\right)$
$= -1$

Therefore, the lines are perpendicular.

b. Textbook questions 11, 14, and 16 of "Practice," p. 294

11. $-\frac{1}{3}$ $\qquad$ **14.** $\frac{2}{5}$ $\qquad$ **16.** 0

c. Textbook questions 28 and 34 of "Practice," p. 294

28.
$$\begin{aligned} m_1 &= m_2 \\ \frac{2}{1} &= -\frac{6}{m} \\ 2(m) &= -6(1) \quad \leftarrow \text{Cross multiply.} \\ 2m &= -6 \\ \frac{2m}{2} &= \frac{-6}{2} \\ m &= -3 \end{aligned}$$

34.
$$\begin{aligned} m_1 \times m_2 &= -1 \\ \frac{1}{3}\left(\frac{m}{6}\right) &= -1 \\ \frac{m}{2} &= -1 \\ 2\left(\frac{m}{2}\right) &= 2(-1) \\ m &= -2 \end{aligned}$$

d. Textbook questions 40 to 42 of "Practice," p. 294

40. Determine the slope of each line.

$$x-y+1=0$$
$$x+1=y$$
$$y=x+1$$
$$y=m_1x+b$$

$$4x+4y+1=0$$
$$4y+1=-4x$$
$$4y=-4x-1$$
$$\frac{4y}{4}=\frac{-4x-1}{4}$$
$$y=-x-\frac{1}{4}$$
$$y=m_2x+b$$

$\therefore\ m_1=1$ and $m_2=-1$

The lines are perpendicular because $m_1\times m_2=1(-1)=-1$.

41. Determine the slope of each line.

$$3x-2y+12=0$$
$$3x+12=2y$$
$$2y=3x+12$$
$$\frac{2y}{2}=\frac{3x+12}{2}$$
$$y=\frac{3}{2}x+6$$
$$y=m_1x+b$$

$$-2x-3y-12=0$$
$$-2x-12=3y$$
$$3y=-2x-12$$
$$\frac{3y}{3}=\frac{-2x-12}{3}$$
$$y=-\frac{2}{3}x-4$$
$$y=m_2x-b$$

$\therefore\ m_1=\frac{3}{2}$ and $m_2=-\frac{2}{3}$

The lines are perpendicular because $m_1\times m_2=\frac{3}{2}\left(-\frac{2}{3}\right)=-1$.

Section 1: Activity 4 (continued)

42. Determine the slope of each line.

$$\begin{aligned} 2x+5y-13&=0\\ 5y-13&=-2x\\ 5y&=-2x+13\\ \frac{5y}{5}&=\frac{-2x+13}{5}\\ y&=-\frac{2}{5}x+\frac{13}{5}\\ y&=m_1x+b \end{aligned} \qquad \begin{aligned} 2x-5y+23&=0\\ 2x+23&=5y\\ 5y&=2x+23\\ \frac{5y}{5}&=\frac{2x+23}{5}\\ y&=\frac{2}{5}x+\frac{23}{5}\\ y&=m_2x+b \end{aligned}$$

$\therefore\ m_1=-\frac{2}{5}$ and $m_2=\frac{2}{5}$

The lines are not parallel because $m_1 \neq m_2$. Also, the lines are not perpendicular because $m_1 \times m_2 = -\frac{2}{5}\left(\frac{2}{5}\right)=-\frac{4}{25}$.

e. Textbook questions 44 and 45 of "Practice," p. 294

44. First, find the slope of the given line.

$$\begin{aligned} 2x-3y+1&=0\\ 2x+1&=3y\\ 3y&=2x+1\\ \frac{3y}{3}&=\frac{2x+1}{3}\\ y&=\frac{2}{3}x+\frac{1}{3}\\ y&=mx+b \end{aligned}$$

$\therefore\ m=\frac{2}{3}$

Because the required line is parallel to the given line, the slope of the required line is also $\frac{2}{3}$.

Next, use the point-slope form to find the equation of the line that passes through point $(1, 2)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= \frac{2}{3}(x - 1) \\ y - 2 &= \frac{2}{3}x - \frac{2}{3} \\ 3(y - 2) &= 3\left(\frac{2}{3}x - \frac{2}{3}\right) \\ 3y - 6 &= 3\left(\frac{2}{3}x\right) - 3\left(\frac{2}{3}\right) \\ 3y - 6 &= 2x - 2 \\ 2x - 2 &= 3y - 6 \\ 2x - 3y + 4 &= 0 \end{aligned}$$

45. First, find the slope of $x - 5y + 2 = 0$.

$$\begin{aligned} x - 5y + 2 &= 0 \\ x + 2 &= 5y \\ 5y &= x + 2 \\ \frac{5y}{5} &= \frac{x + 2}{5} \\ y &= \frac{1}{5}x + \frac{2}{5} \\ y &= mx + b \end{aligned}$$

$\therefore\ m = \frac{1}{5}$

Because the slope of the required line is perpendicular to the given line, the slope of the required line is -5, since $\frac{1}{5}(-5) = -1$.

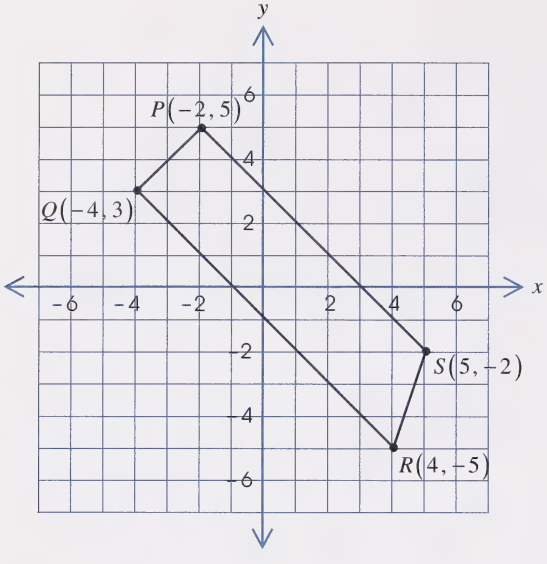
Next, use the point-slope form to determine the equation of the line that passes through $(-2, 5)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= -5[x - (-2)] \\ y - 5 &= -5(x + 2) \\ y - 5 &= -5x - 10 \\ 5x + y + 5 &= 0 \end{aligned}$$

Section 1: Activity 4 (continued)

f. Textbook questions 53, 59, and 61 of "Applications and Problem Solving," p. 295

53.



To classify quadrilateral *PQRS*, determine the slope of each side.

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{-4-(-2)} = \frac{-2}{-2} = 1$$

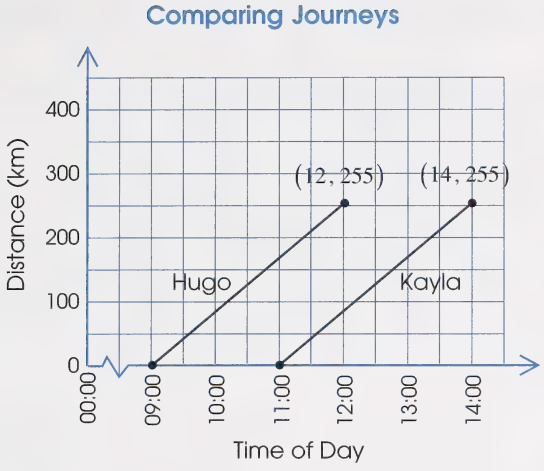
$$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-5)}{5-4} = \frac{3}{1} = 3$$

$$m_{QR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-3}{4-(-4)} = \frac{-8}{8} = -1$$

$$m_{PS} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-5}{5-(-2)} = \frac{-7}{7} = -1$$

Only two sides are parallel: *QR* and *PS* $\left(m_{QR} = m_{PS}\right)$. Therefore, *PQRS* is a trapezoid.

59. a.



Because Hugo drove 255 km at 85 km/h, his journey took $\frac{255 \text{ km}}{85 \text{ km/h}} = 3$ h. He started his journey at 09:00 and arrived in Crowsnest Pass at 12:00.

Kayla drove the same 255 km at 85 km/h. Her journey also took 3 h. However, she started at 11:00 and arrived in Crowsnest Pass at 14:00.

b. The lines are parallel because the slopes, given by their speeds (85 km/h), are equal.

61. a. Find the slope of each line.

$$3x - 2y - 5 = 0$$
$$3x - 5 = 2y$$
$$2y = 3x - 5$$
$$\frac{2y}{2} = \frac{3x - 5}{2}$$
$$y = \frac{3}{2}x - \frac{5}{2}$$
$$y = m_1 x + b$$

$$kx - 6y + 1 = 0$$
$$kx + 1 = 6y$$
$$6y = kx + 1$$
$$\frac{6y}{6} = \frac{kx + 1}{6}$$
$$y = \frac{k}{6}x + \frac{1}{6}$$
$$y = m_2 x + b$$

$\therefore m_1 = \frac{3}{2}$ and $m_2 = \frac{k}{6}$

Section 1: Activity 4 (continued)

Because the lines are parallel,

$$m_1 = m_2$$
$$\frac{3}{2} = \frac{k}{6}$$
$$6(3) = 2(k) \quad \leftarrow \text{Cross multiply.}$$
$$18 = 2k$$
$$2k = 18$$
$$\frac{2k}{2} = \frac{18}{2}$$
$$k = 9$$

b. Because the lines are perpendicular,

$$m_1 \times m_2 = -1$$
$$\frac{1}{\cancel{3}_{2}} \times \left(\frac{k}{\cancel{6}_{2}} \right) = -1$$
$$\frac{k}{4} = -1$$
$$4\left(\frac{k}{4} \right) = 4(-1)$$
$$k = -4$$

3. Textbook question 1 of "Data Bank," p. 303

1. a. It is any time you want it to be at the North Pole; the 24 time zones converge at the poles. Normally, the time explorers use is the time at their base camp.

b. The time-zone map is misleading because the time zones are not shown converging at the poles.

Section 1: Activity 5

1. a. Textbook questions a. to d. of "Explore: Draw a Graph," p. 296

a. If $n = 0$, then

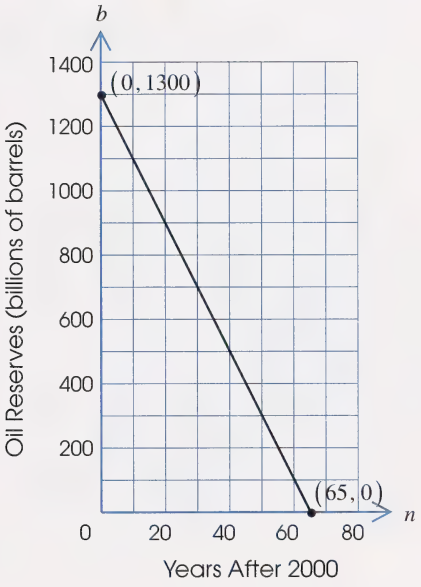
$$b + 20n = 1300$$
$$b + 20(0) = 1300$$
$$b = 1300$$

b. If $b = 0$, then

$$b + 20n = 1300$$
$$0 + 20n = 1300$$
$$\frac{20n}{20} = \frac{1300}{20}$$
$$n = 65$$

c. The ordered pairs in the form (n, b) are $(65, 0)$ and $(0, 1300)$.

d.



b. Textbook questions 1 to 6 of "Inquire," p. 296

1. The b-intercept represents the world's oil reserves in the year 2000.

2. The n-intercept represents the number of years after 2000 before the world's oil reserves run out.

3. In 2030, the oil reserves appear to be 700 billion barrels. In 2045, the oil reserves appear to be 400 billion barrels.

Section 1: Activity 5 (continued)

4. Let $(n_1, b_1) = (0, 1300)$ and $(n_2, b_2) = (65, 0)$.

$$\text{slope} = \frac{b_2 - b_1}{n_2 - n_1}$$

$$m = \frac{0 - 1300}{65 - 0}$$

$$= \frac{-1300}{65}$$

$$= -20 \text{ billion barrels / year}$$

5. The slope represents the annual decline in the world's oil reserves.

6. Some conditions that might change the slope are as follows:

- conservation measures
- development of alternate energy sources
- changing population levels

2. Textbook questions 1, 3, 5, 9, 13, and 14 of "Practice," p. 298

1. To find the x-intercept, let $y = 0$.

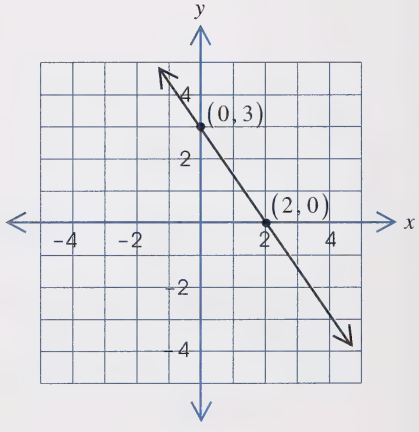
$$3x + 2(0) = 6$$

$$3x = 6$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

One point on the line is $(2, 0)$.



To find the y-intercept, let $x = 0$.

$$3(0) + 2y = 6$$

$$2y = 6$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

Another point on the line is $(0, 3)$.

3. To find the x-intercept, let $y = 0$.

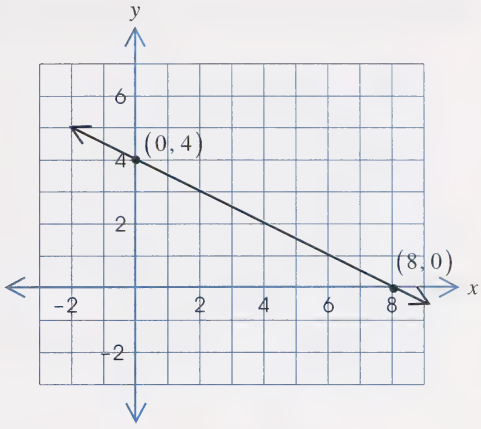
$$x + 2(0) = 8$$
$$x = 8$$

One point on the line is $(8, 0)$.

To find the y-intercept, let $x = 0$.

$$0 + 2y = 8$$
$$2y = 8$$
$$\frac{2y}{2} = \frac{8}{2}$$
$$y = 4$$

Another point on the line is $(0, 4)$.



5. To find the x-intercept, let $y = 0$.

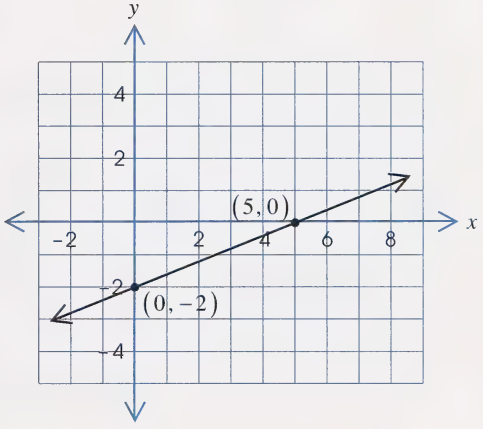
$$2x - 5(0) = 10$$
$$2x = 10$$
$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

One point on the line is $(5, 0)$.

To find the y-intercept, let $x = 0$.

$$2(0) - 5y = 10$$
$$-5y = 10$$
$$\frac{-5y}{-5} = \frac{10}{-5}$$
$$y = -2$$

Another point is $(0, -2)$.



Section 1: Activity 5 (continued)

9. $y = 3x - 2$

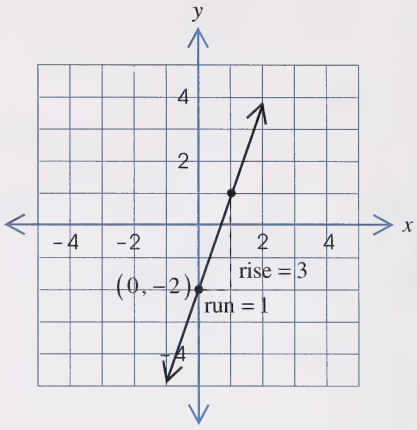
$y = mx + b$

$\therefore\ m = 3$ and $b = -2$

Plot the point $(0, -2)$.

Because $m = 3 = \frac{3}{1}$, count 1 unit to the right and 3 units up from $(0, -2)$.

Finally, draw a line through the two points.



13. $2x + y = 0$

$y = -2x$

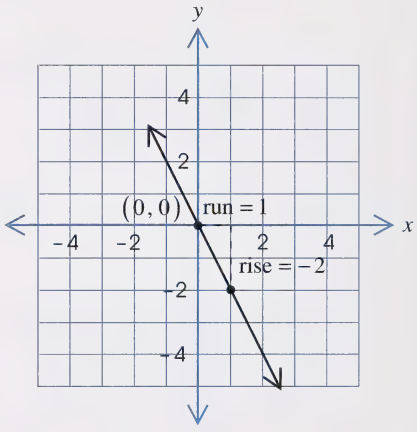
$y = mx + b$

$\therefore\ m = -2$ and $b = 0$

Plot the point $(0, 0)$.

Because $m = -2 = \frac{-2}{1}$, count 1 unit to the right and 2 units down from $(0, 0)$.

Finally, draw a line through the two points.



14. $3x+2y-6=0$

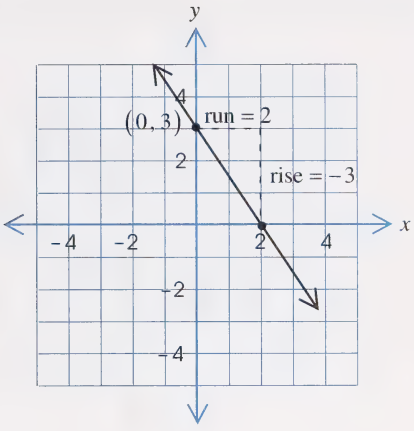
$$2y=-3x+6$$

$$\frac{2y}{2}=\frac{-3x+6}{2}$$

$$y=-\frac{3}{2}x+3$$

$$y=mx+b$$

$\therefore\ m=-\frac{3}{2}$ and $b=3$

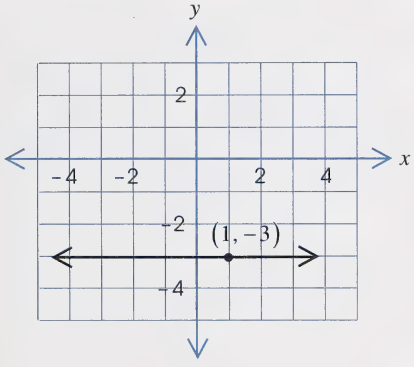


Plot the point $(0,3)$.

Because $m=-\frac{3}{2}$, count 2 units to the right and 3 units down from $(0,3)$.

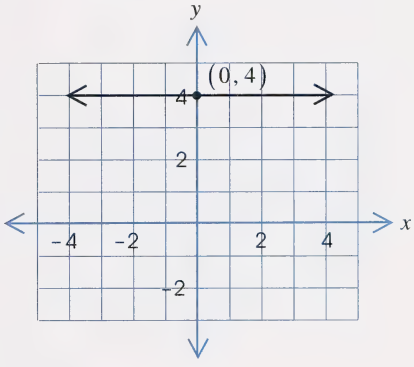
Finally, draw a line through the two points.

3. a.



$y=-3$ or $y+3=0$

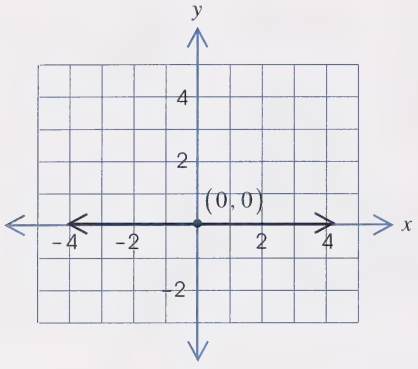
b.



$y=4$ or $y-4=0$

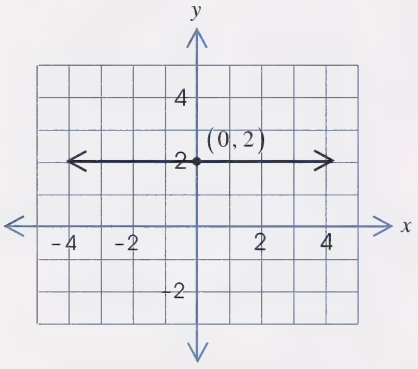
Section 1: Activity 5 (continued)

c.



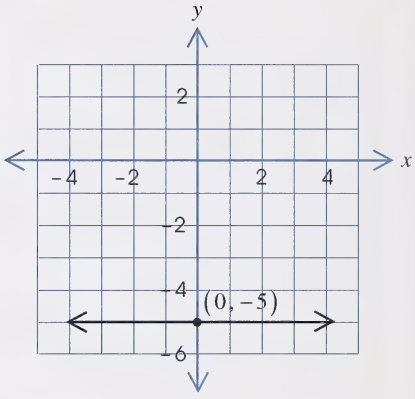
$y = 0$

4. a. $y - 2 = 0$ may be written as $y = 2$.



The y-intercept of this horizontal line is 2.

b.



$y = -5$ passes through $(0, -5)$.
Therefore, the y-intercept is -5.

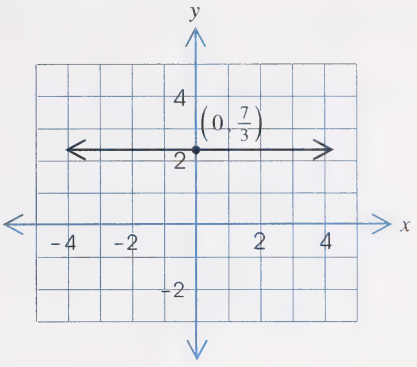
c. $3y-7=0$

$3y=7$

$y=\frac{7}{3}$

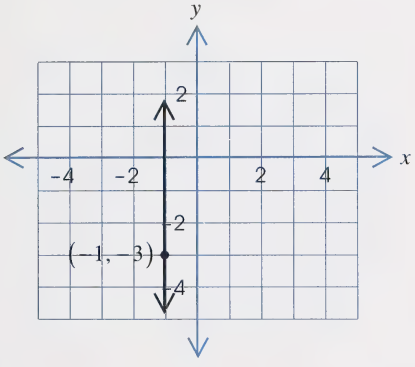
This horizontal line passes through $\left(0,\frac{7}{3}\right)$ or $\left(0,2\frac{1}{3}\right)$.

Therefore, the y-intercept is $\frac{7}{3}$ or $2\frac{1}{3}$.



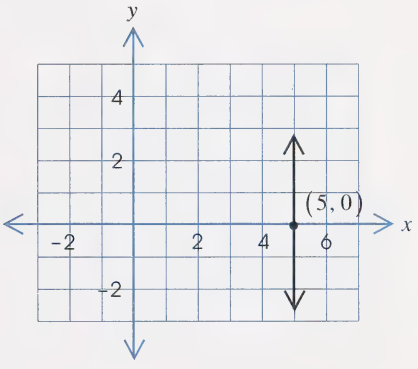
5. The equation of the x-axis is $y=0$.

6. a.



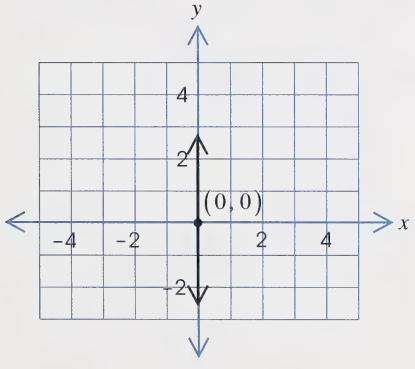
$x=-1$ or $x+1=0$

b.



$x=5$ or $x-5=0$

c.



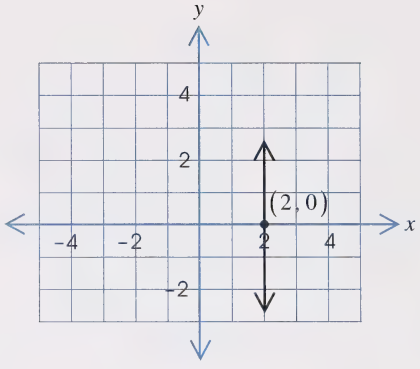
$x=0$

Section 1: Activity 5 (continued)

7. a. $x-2=0$

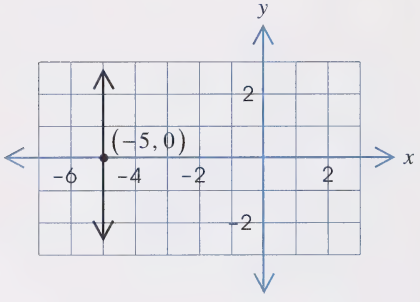
$x=2$

The x-intercept of this vertical line is 2.



b. $x=-5$ passes through $(-5,0)$.

Therefore, the x-intercept is -5.

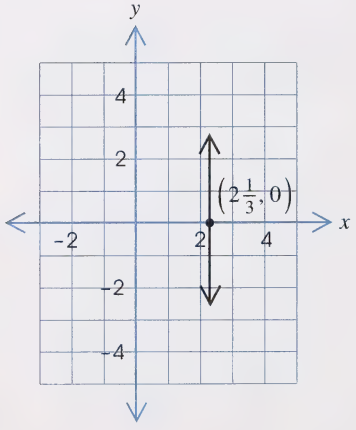


c. $3x-7=0$

$3x=7$

$x=\frac{7}{3}$ or $2\frac{1}{3}$

Therefore, the x-intercept is $\frac{7}{3}$ or $2\frac{1}{3}$.



8. The equation of the y-axis is $x=0$.

9. a. Textbook questions 26 and 28 of "Practice," p. 298

26. $y = 3$ **28.** $x = 2$

b. Textbook questions 29 to 32 of "Applications and Problem Solving," p. 298

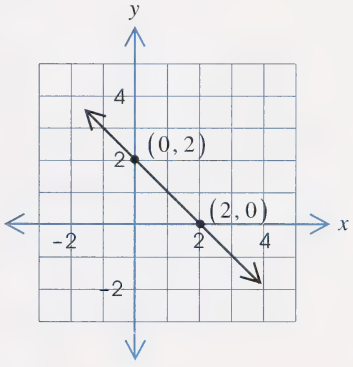
29. Answers will vary. A sample answer is given.

Any equation of the form $y = b$, where $b \neq 0$, represents a horizontal line with no x-intercept. For instance, $y = 4$.

30. Answers will vary. A sample answer is given.

Any equation of the form $x = a$, where $a \neq 0$, represents a vertical line with no y-intercept. For instance, $x = -2$.

31. Answers will vary. A sample answer is given.



$x + y - 2 = 0$ has equal intercepts.

If $y = 0$, then $x + 0 - 2 = 0$

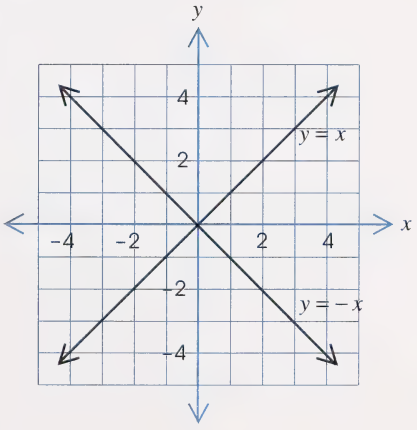
$x = 2$

If $x = 0$, then $0 + y - 2 = 0$

$y = 2$

The x-intercept is 2, and the y-intercept is 2.

32. There are many possible equations. Two sample equations are $y = x$ and $y = -x$.



The x-intercept = 0, and the y-intercept = 0.

Section 1: Activity 5 (continued)

10. Textbook questions 1 to 6 of Investigation 1, "Antarctic Ice," p. 290

1. $3\,000\,000\ \text{km}^2$

2. a. The area at day 60 is $3\,000\,000\ \text{km}^2$. The area at day 240 is $21\,000\,000\ \text{km}^2$. Therefore, the increase is $18\,000\,000\ \text{km}^2$.

b. The increase is about 1.8 times (almost double) the area of Canada.

3. a. Let $(x_1, y_1) = (60, 3\,000\,000)$ and $(x_2, y_2) = (240, 21\,000\,000)$.

$$\therefore\ m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{21\,000\,000 - 3\,000\,000}{240 - 60}$$
$$= \frac{18\,000\,000}{180}$$
$$= 100\,000$$

The slope of the line segment between day 60 and day 240 is $100\,000\ \text{km}^2/\text{day}$.

b. The slope represents the rate at which the sea ice increases every day.

c. $100\,000\ \text{km}^2/\text{day} = 100\,000\ \text{km}^2/24\ \text{h}$
$\doteq 4166.7\ \text{km}^2/\text{h}$
$\doteq 4166.7\ \text{km}^2/60\ \text{min}$
$\doteq 69.4\ \text{km}^2/\text{min}$

4. The amount of sea ice remains constant.

5. The amount of sea ice decreases from $21\,000\,000\ \text{km}^2$ to $3\,000\,000\ \text{km}^2$.

6. a. Let $(x_1, y_1) = (270, 21\,000\,000)$ and $(x_2, y_2) = (360, 3\,000\,000)$.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3\,000\,000 - 21\,000\,000}{360 - 270}$$
$$= \frac{-18\,000\,000}{90}$$
$$= -200\,000$$

The slope of the line segment between day 270 and day 360 is $-200\,000$ km^2 / day.

b. The slope represents the decrease in the area of sea ice per day.

c. $-200\,000$ km^2 / day $= -200\,000$ km^2 / 24 h
$\doteq -8333.3$ km^2 / h
$\doteq -8333.3$ km^2 / 60 min
$\doteq -138.9$ km^2 / min

Section 1: Follow-up Activities

Extra Help

1. Method 1: Using the Point-Slope Formula

First, find the slope of the line. Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (1, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{1-2}{1-(-1)}$$
$$= \frac{-1}{2}$$
$$= -\frac{1}{2}$$

Extra Help (continued)

Now, substitute $m=-\frac{1}{2}$ and $(x_1, y_1)=(-1, 2)$ into the point-slope formula.

$$\begin{aligned}
y-y_1 &= m(x-x_1) \\
y-2 &= -\frac{1}{2}[x-(-1)] \\
y-2 &= -\frac{1}{2}(x+1) \\
y-2 &= -\frac{1}{2}x-\frac{1}{2} \\
2(y-2) &= 2\left(-\frac{1}{2}x-\frac{1}{2}\right) \\
2y-4 &= -x-1 \\
x+2y-3 &= 0
\end{aligned}$$

Method 2: Using the Slope and *y*-Intercept Formula

Because $(-1, 2)$ is on the line, find b by substituting $m=-\frac{1}{2}$ and $(-1, 2)$ into the slope and y-intercept formula.

$$\begin{aligned}
y &= mx+b \\
2 &= -\frac{1}{2}(-1)+b \\
2 &= \frac{1}{2}+b \\
\frac{1}{2}+b &= 2 \\
b &= 1\frac{1}{2} \text{ or } \frac{3}{2}
\end{aligned}$$

Substitute $m=-\frac{1}{2}$ and $b=\frac{3}{2}$ into the slope and y-intercept formula.

$$\begin{aligned}
y &= mx+b \\
y &= -\frac{1}{2}x+\frac{3}{2} \\
2(y) &= 2\left(-\frac{1}{2}x+\frac{3}{2}\right) \\
2y &= -x+3 \\
x+2y-3 &= 0
\end{aligned}$$

2. Method 1: Using the Slope and *y*-Intercept Formula

Substitute $m = -2$ and $b = 6$ into the slope and *y*-intercept formula.

$$\begin{aligned} y &= mx + b \\ y &= -2x + 6 \\ 2x + y - 6 &= 0 \end{aligned}$$

Method 2: Using the Point-Slope Formula

Because the *y*-intercept is 6, the line passes through the *y*-axis at $(0, 6)$. Thus, substitute $m = -2$ and $(x_1, y_1) = (0, 6)$ into the point-slope formula.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 6 &= -2(x - 0) \\ y - 6 &= -2x \\ 2x + y - 6 &= 0 \end{aligned}$$

Enrichment

1. Textbook questions 1 and 2 of Investigation 1, "Distance Between Two Points," p. 274

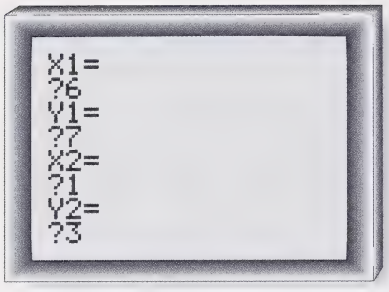
1.

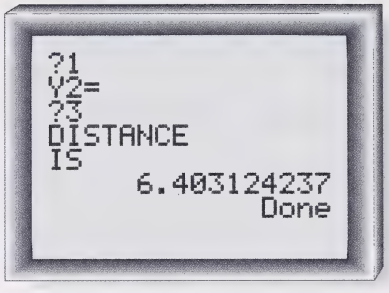
Program Line	What Each Line Does
PROGRAM:DISTANCE	names the program
:ClrHome	clears the memory
:Disp "X1="	asks for x_1
:Input P	stores x_1 as variable *P*
:Disp "Y1="	asks for y_1
:Input Q	stores y_1 as variable *Q*
:Disp "X2="	asks for x_2
:Input R	stores x_2 as variable *R*
:Disp "Y2="	asks for y_2
:Input S	stores y_2 as variable *S*
:R–P→X	assigns R–P to variable X
:S–Q→Y	assigns S–Q to variable Y
:$\sqrt{}$ (X²+Y²) →D	calculates the distance and assigns it to variable *D*
:Disp "DISTANCE"	displays "DISTANCE"
:Disp "IS"	displays "IS"
:Disp D	displays the distance

Enrichment (continued)

2. The answers given here show two calculator displays. This is to show you all of the information entered and the results obtained.

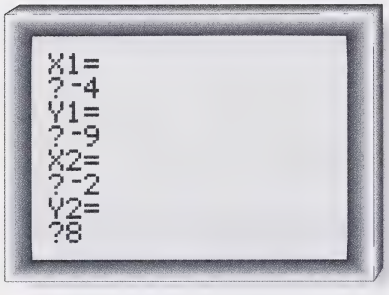
a.

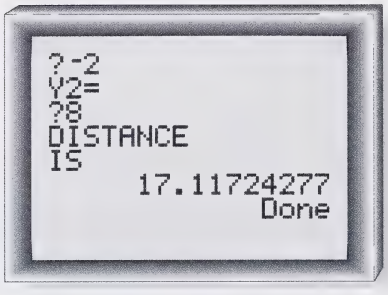




The distance, d, is approximately 6.4.

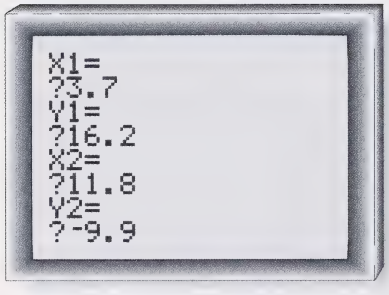
b.

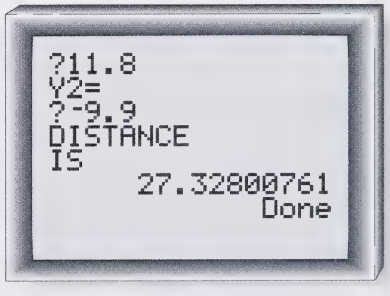




The distance, d, is approximately 17.1.

c.





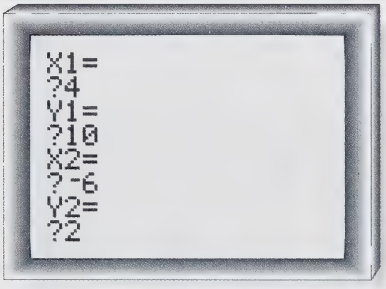
The distance, d, is approximately 27.33.

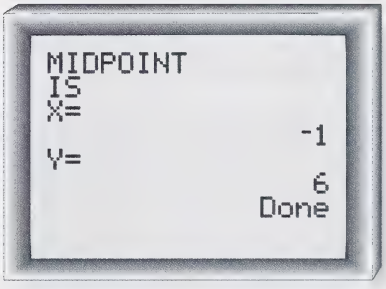
2. Textbook questions 1 and 2 of Investigation 2, "Midpoint of a Line Segment," p. 274

1. PROGRAM:MIDPOINT
:ClrHome
:Disp "X1="
:Input P
:Disp "Y1="
:Input Q
:Disp "X2="
:Input R
:Disp "Y2="
:Input S
:(P+R)/2→X
:(Q+S)/2→Y
:Disp "MIDPOINT"
:Disp "IS"
:Disp "X="
:Disp X
:Disp "Y="
:Disp Y

2. The answers given here show two calculator displays. This is to show you all of the information entered and the results obtained.

a.

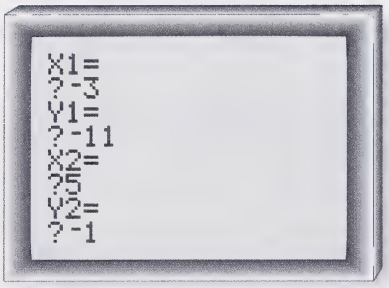


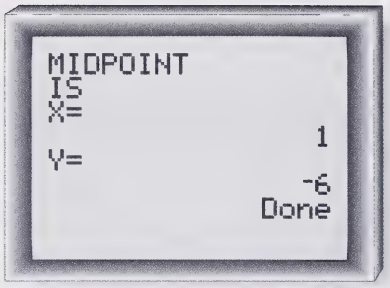


The midpoint is $(-1, 6)$.

Enrichment (continued)

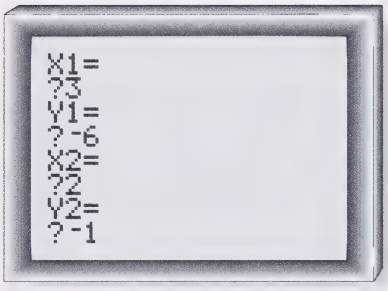
b.

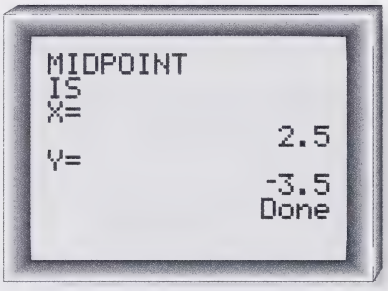




The midpoint is $(1, -6)$.

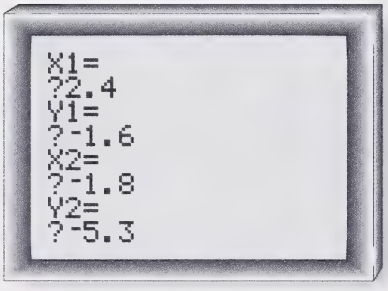
c.

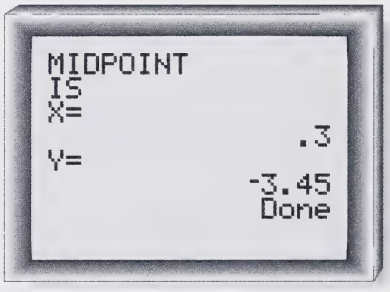




The midpoint is $(2.5, -3.5)$.

d.





The midpoint is $(0.3, -3.45)$.

Section 2: Activity 1

1. a. Textbook question "Explore: Look for a Pattern," p. 212

Side Length of Large Square(s)	Number of Red Squares (n)	Ordered Pair (s, n)
3	8	$(3, 8)$
4	12	$(4, 12)$
5	16	$(5, 16)$
6	20	$(6, 20)$
7	24	$(7, 24)$

b. Textbook questions 1 to 5 of "Inquire," p. 212

1. The greatest common factor of the number of red squares is 4.

2. $n = 4s - 4$, where $s = 3, 4, 5, 6, 7, \ldots$

3. To find the number of red squares, n, multiply the side length, s, by 4. Then subtract 4 from the product.

4. If $s = 14$, then $n = 4s - 4$

$= 4(14) - 4$

$= 56 - 4$

$= 52$

If $s = 40$, then $n = 4s - 4$

$= 4(40) - 4$

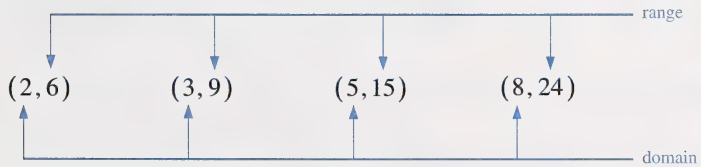
$= 160 - 4$

$= 156$

5. $(8, 28)$ means that a square with a side length of 8 has 28 red squares that form the border.

2. a. Textbook questions 1 and 3 of "Practice," p. 214

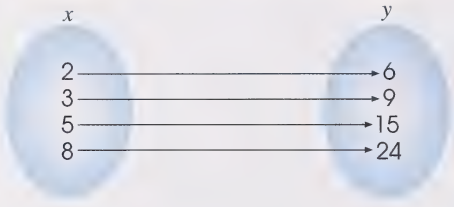
1. a.



The domain is $\{2, 3, 5, 8\}$, and the range is $\{6, 9, 15, 24\}$.

Section 2: Activity 1 (continued)

b.



c. Each element in the range is 3 times the corresponding element in the domain.

3. a. The domain is $\{4, 5, 7, 21\}$.
The range is $\{9\}$.

b.

c. The only element in the range is 9.

b. Textbook questions 5 and 7 of "Practice," p. 214

5. a. $(0,1),\ (1,1),\ (2,5),\ (3,6)$

b. The domain is $\{0, 1, 2, 3\}$, and the range is $\{1, 5, 6\}$.

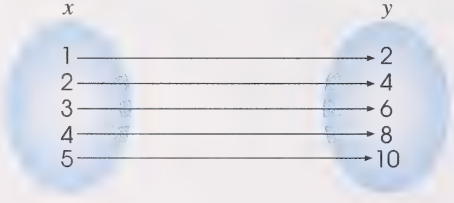
Note: Elements are normally arranged in ascending order.

7. a. $(-2,0),\ (-2,1),\ (0,2),\ (3,2),\ (3,3)\ (5,3),\ (5,4)$

b. The domain is $\{-2, 0, 3, 5\}$, and the range is $\{0, 1, 2, 3, 4\}$.

c. Textbook questions 9, 11, and 13 of "Practice," p. 214

9.



11. Answers will vary. A sample answer is given.

$(-2,-8),\ (-1,-4),\ (0,0),\ (1,4),\ (2,8)$

13. a. $(-4,4),\ (-3,4),\ (-2,4),\ (-1,3),\ (0,2),\ (1,1),\ (2,0)$

b. The domain is $\{-4,-3,-2,-1,0,1,2\}$, and the range is $\{4,3,2,1,0\}$ or $\{0,1,2,3,4\}$.

d. Textbook questions 17 and 18 of "Applications and Problem Solving," pp. 214 and 215

17.

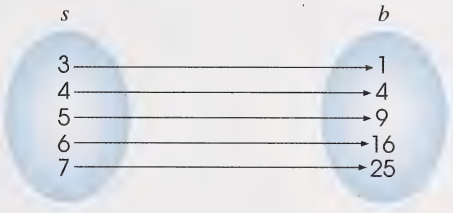
Side Length of Large Square (s)	Number of Blue Squares (b)
3	1
4	4
5	9
6	16
7	25

a. The number of blue squares is $1^2,\ 2^2,\ 3^2,\ 4^2,\ 5^2,\ \ldots$.

b. $(3,1),\ (4,4),\ (5,9),\ (6,16),\ (7,25)$

c. The domain is $\{3,4,5,6,7\}$, and the range is $\{1,4,9,16,25\}$.

d.



e. The number of blue squares is found by subtracting 2 from the side length of the large square and then squaring the difference.

f. $b=(s-2)^2$, where $s=3,\ 4,\ 5,\ 6,$ and 7

Section 2: Activity 1 (continued)

18. **a.** $(140, 70)$, $(150, 75)$, $(160, 80)$, $(170, 85)$, $(180, 90)$

b. The domain is $\{140, 150, 160, 170, 180\}$, and the range is $\{70, 75, 80, 85, 90\}$.

c.

d. The length of the mirror is half the height of the person.

e. $\ell = \frac{1}{2}h$ or $\ell = \frac{h}{2}$, where $h = 140$, 150, 160, 170, and 180

3. Textbook questions 1 to 10 of "Mental Math: Expressions and Equations," p. 209

1. 3, 7, 11, 19, 25

2. 8, 11, 20, 32, 59

3. 7, 6, 5, 3, 0

4. -7, -12, -17, -27, -52

5. -4, -2, 10, 12, 14

6. 19, 7, 3, 7, 19

7. 19, 3, -5, -5, 3

8. Answers will vary. A sample answer is given.

$(1, 7)$, $(2, 6)$, $(3, 5)$, $(4, 4)$

9. Answers will vary. A sample answer is given.

$(5, 3)$, $(4, 2)$, $(3, 1)$, $(2, 0)$

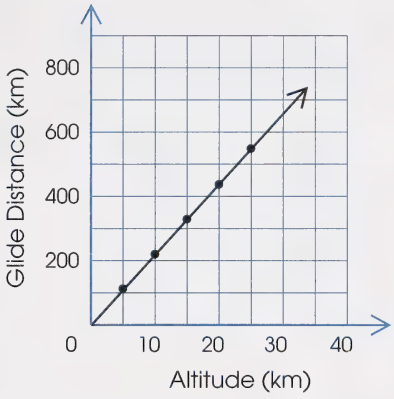
10. Answers will vary. A sample answer is given.

$(0, -1)$, $(-1, 0)$, $(-2, 1)$, $(1, -2)$

Section 2: Activity 2

1. a. Textbook question "Explore: Draw a Graph," p. 216

Altitude (km)	Glide Distance (km)
5	110
10	220
15	330
20	440
25	550

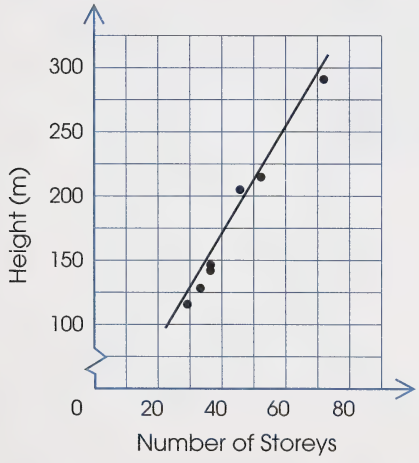


b. Textbook questions 1 and 2 of "Inquire," p. 216

1. This is called a linear relation because the points of the graph lie along a straight line.

2. a. From an altitude of 18 km, the plane glides 396 km.
b. From an altitude of 30 km, the plane glides 660 km.

2. Textbook question "Explore: Draw a Graph," p. 217

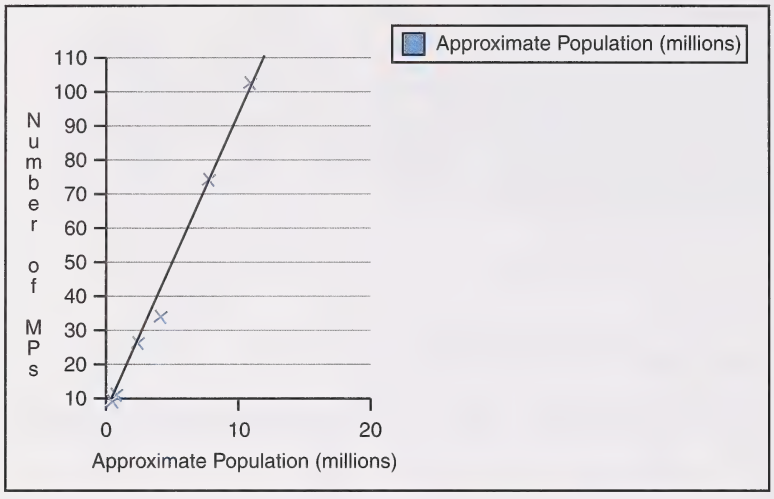


Section 2: Activity 2 (continued)

3. Textbook questions 1, 2.a., and 3 of Investigation 1, "Members of Parliament," pp. 222 and 223

1. The number of MPs elected from each province appears to be based on the percentage of Canada's population that resides in each province. It appears to be fair, based on population.

2. **a.**



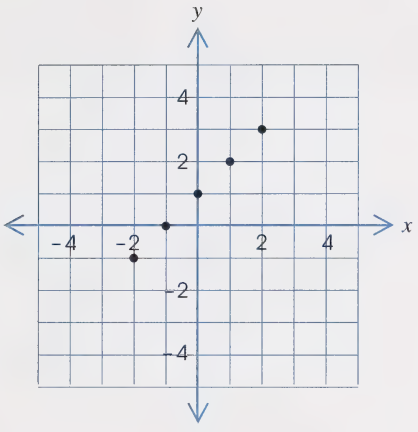
3.

Province	Approximate Population (millions)	Number of MPs
Manitoba	1.1	15
Newfoundland	0.6	7
PEI	0.4	4
Saskatchewan	1.0	14

4. a. Textbook questions 1 and 3 of "Practice," p. 219

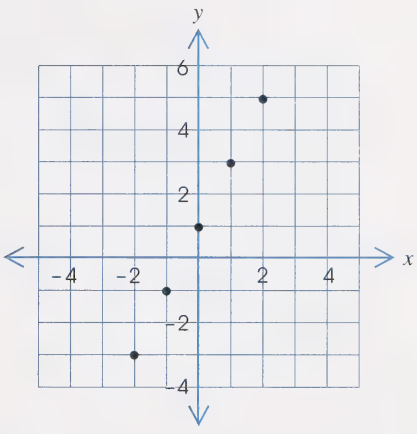
1. $y = x + 1$

x	2	1	0	−1	−2
y	3	2	1	0	−1



3. $y = 2x + 1$

x	2	1	0	−1	−2
y	5	3	1	−1	−3



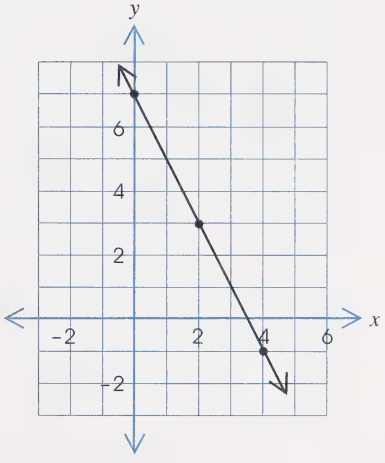
b. Textbook questions 12 and 19 of "Practice," p. 219

12. $y = -2x + 7$

Choose values of x that give convenient values of y. Complete a table of values.

x	0	2	4
y	7	3	−1

Plot the points. Because the domain is R, the graph is continuous. Therefore, join the points with a line.



Section 2: Activity 2 (continued)

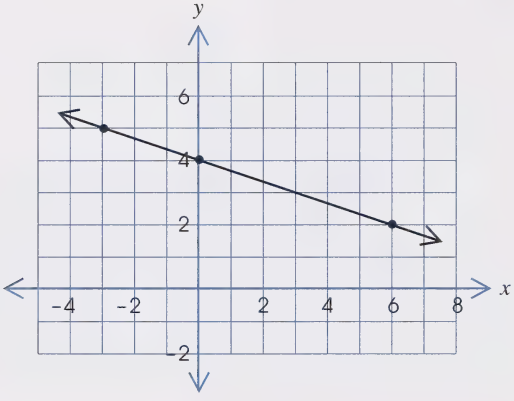
19. Solve the equation for y.

$$x+3y-12=0$$
$$3y=12-x$$
$$y=\frac{12-x}{3}$$

Choose values of x that give convenient values of y. Complete a table of values.

x	0	6	−3
y	4	2	5

Plot the points. Because the domain is R, the graph is continuous. Therefore, join the points with a line.



c. Textbook question 24 of "Practice," p. 219

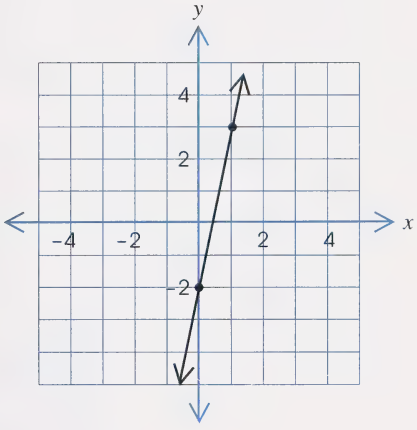
24. Solve for y.

$$5x - y - 2 = 0$$
$$5x - 2 = y$$
$$y = 5x - 2$$

Choose convenient values of x, and solve for y. Complete a table of values.

x	0	1
y	-2	3

Plot the points. Again, because the domain is R, join the points with a line.



To obtain x-values from y-values, solve the equation for x.

$$5x - y - 2 = 0$$
$$5x = y + 2$$
$$x = \frac{y+2}{5}$$

If $y = 1.3$, then $x = \frac{1.3+2}{5}$

$$= 0.66$$
$$\doteq 0.7$$

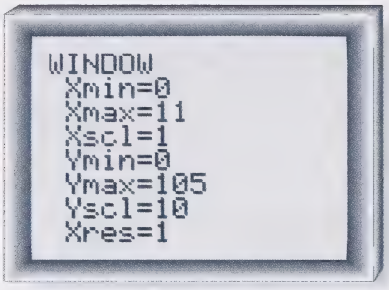
If $y = -2.1$, then $x = \frac{-2.1+2}{5}$

$$= \frac{-0.1}{5}$$
$$= -0.02$$
$$\doteq 0.0$$

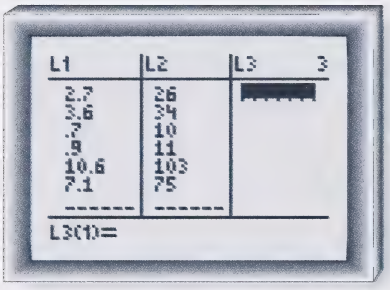
Section 2: Activity 2 (continued)

5. Textbook question 5.a. of Investigation 1, "Members of Parliament," p. 223

5. a. To graph the scatter plot and the line of best fit on a graphing calculator, first set the graphing window to appropriate values. For instance, press [WINDOW] and enter the following values:



Next, press [STAT] [1] and enter the following lists from the table given in the question.

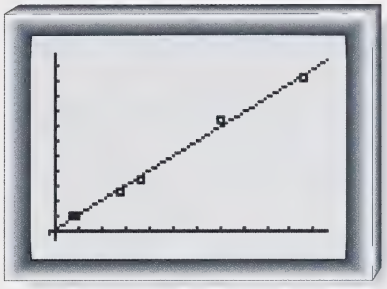


Before displaying the scatter plot, press [2nd] [STAT PLOT] [1]. Check to see if Plot1 is on and that the graph type chosen is a scatter plot (the first type shown). Select them if they are not highlighted. Use the arrow keys to move the cursor, and press [ENTER] after you have highlighted your choice.

Now, press [ZOOM] [9]. This displays the scatter plot.

Now, draw the line of best fit. First, press [STAT], choose the CALC menu, and press [4] to paste "LinReg(ax+b)" on the screen. Next, press [2nd] [1] [,] [2nd] [2] [,] [VARS] [Select the Y-VARS menu.] [1] (to select the FUNCTION menu) [1] (to select "Y_1") [ENTER].

Now, press GRAPH to see the line of best fit.



To complete the table (question 3), you can use the equation of the line of best fit. Press Y=.

You will see $Y_1 = 9.7043678977273X + 1.761363636364$, where Y_1 is the number of MPs and X is the approximate population in millions. Simply substitute the values given and solve for the missing values.

To find the number of MPs for Manitoba, for example, replace X by 1.1 and solve for Y_1.

$$Y_1 \doteq 12.43$$

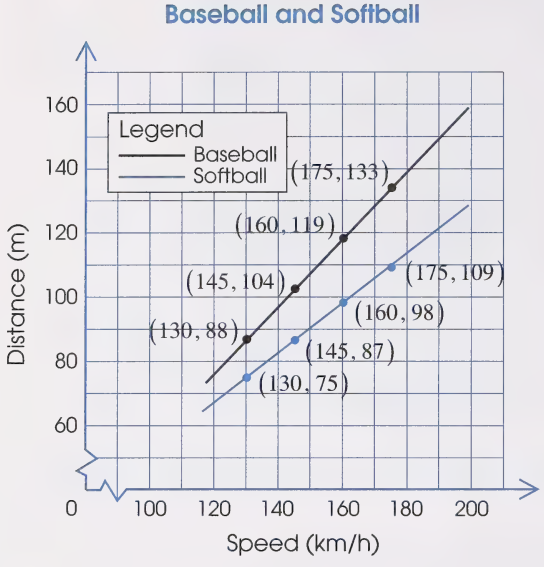
Round to 12.

Province	Approximate Population (millions)	Number of MPs
Manitoba	1.1	12
Newfoundland	0.6	8
PEI	0.2	4
Saskatchewan	1.3	14

Section 2: Activity 2 (continued)

6. Textbook questions 26, 28, and 31 of "Applications and Problem Solving," pp. 219 and 220

26. a.



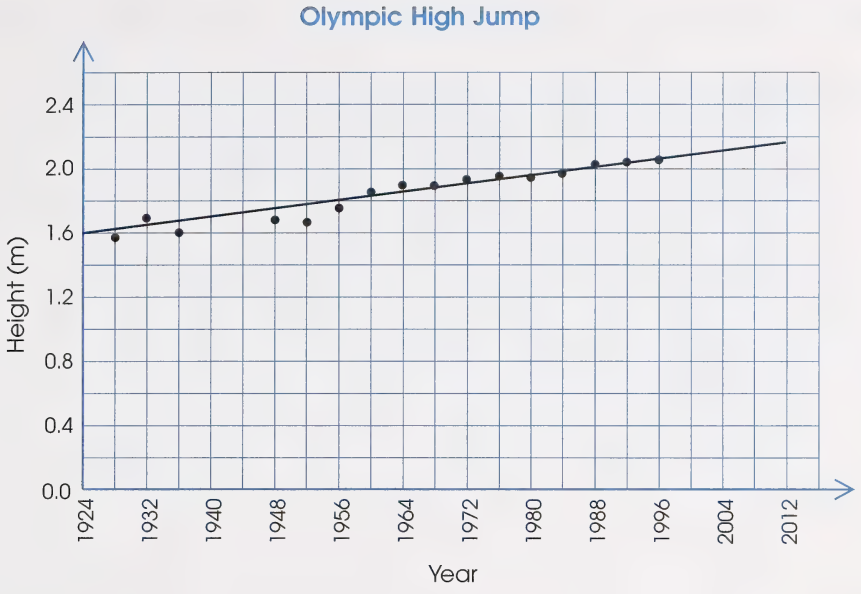
b. Refer to the answer to question 26.a.

c. Answers may vary because lines of best fit may vary slightly. According to this graph, a pitcher can throw a baseball about 93 m.

d. Answers may vary because lines of best fit may vary slightly. According to this graph, the distance travelled by a baseball is about 150 m, and the distance travelled by a softball is about 120 m.

e. The softball does not travel as far as the baseball because the softball experiences greater air resistance.

28. a.



b. Refer to the answer to question 28.a.

c. There were no Olympics during the Second World War. The winning heights may have been about 1.70 m in 1940 and about 1.75 m in 1944.

Note: Heights may vary because the line of best fit may vary slightly.

d. Answers may vary because the line of best fit may vary slightly. For this graph, the winning height in the year 2012 may be about 2.15 m.

e. The answer may not be reasonable because the slope of the graph will eventually decrease to zero.

f. The winning height has increased with time because of changing techniques, improved training, better nutrition, and increased participation in the sport.

g. Yes, there will likely be a maximum height recorded in the high jump. There are limits to what the human body can perform.

31. a. domain = Reals
range = Reals

b. domain = Reals
range = Reals

c. domain = Reals
range = $\{3\}$

d. domain = $\{-2\}$
range = Reals

Section 2: Activity 2 (continued)

7. Textbook question 1 of Investigation 1, "Gold Mining," p. 221

1. a. Brazil uses about $130t$ of mercury every year.

b. The slope of the graph is the rate at which mercury is used to extract a tonne of gold.

Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (90, 130)$.

$$\therefore \text{ slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{130 - 0}{90 - 0}$$
$$\doteq 1.4$$

Approximately 1.4 t of mercury is used in extracting 1 t of gold.

Section 2: Activity 3

1. Textbook questions 1 to 6 of "Inquire," p. 224

1. The tilt angle in 1700 was 5°.

2. The tilt angle was 3° in 1400.

3. The tilt angle increased the fastest between 1325 and 1400.

4. Because the slope of the graph remains positive, the tower continues to lean further. The tilt angle continues to increase, although at a slow rate.

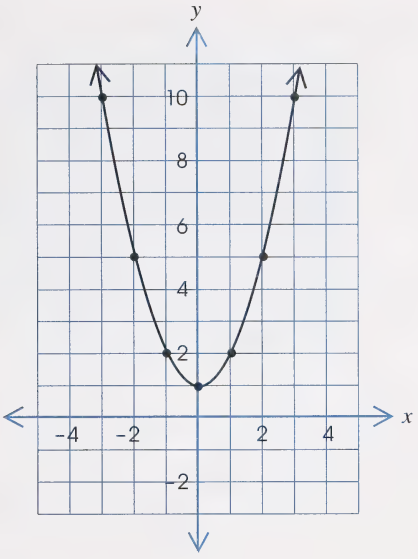
5. The domain is year > 1100, and the range is $0^\circ \leq$ tilt angle $< 90^\circ$.

6. No, the Pisans would not want to push the tower upright because it is a tourist attraction; thus, it would affect their local economy.

2. a. Textbook questions 1 and 3 of "Practice," p. 225

1. $y = x^2 + 1$

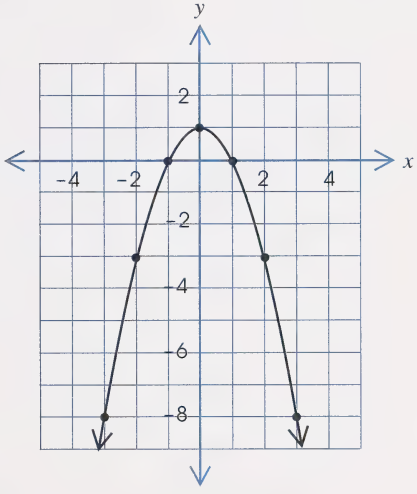
x	y
0	1
1	2
−1	2
2	5
−2	5
3	10
−3	10



The domain is R, and the range is $y \geq 1$.

3. $y = -x^2 + 1$

x	y
0	1
1	0
−1	0
2	−3
−2	−3
3	−8
−3	−8



The domain is R, and the range is $y \leq 1$.

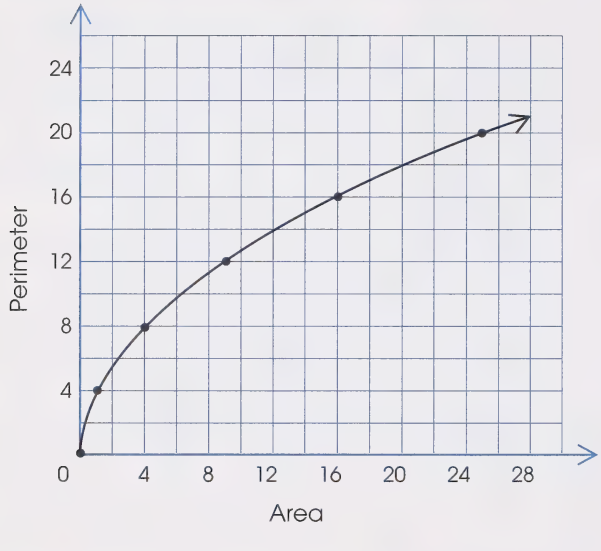
Section 2: Activity 3 (continued)

b. Textbook questions 5, 6, 9, 10, and 13 of "Applications and Problem Solving," pp. 226 and 227

5. a.

Side Length	Area	Perimeter
0	0	0
1	1	4
2	4	8
3	9	12
4	16	16
5	25	20

b.



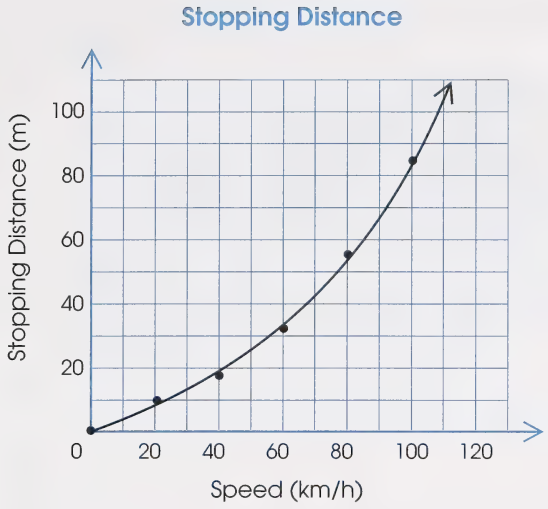
c. The area is about 12 square units.

d. The perimeter is about 18 units.

e. The domain is area ≥ 0, and the range is perimeter ≥ 0.

f. $A = \left(\frac{P}{4}\right)^2$ or $A = \frac{P^2}{16}$

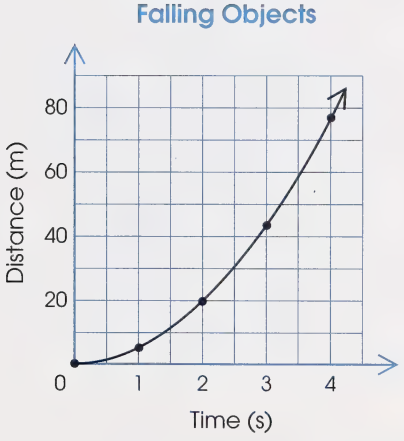
6. a.



b. Answers may vary slightly. At 50 km/h, the stopping distance is approximately 25 m. At 70 km/h, the stopping distance is about 43 m.

c. Answers may vary slightly. A stopping distance of 100 m corresponds to a speed of approximately 107 km/h.

9. a.



b. It takes about 2.7 s to fall 36 m.

Section 2: Activity 3 (continued)

c.

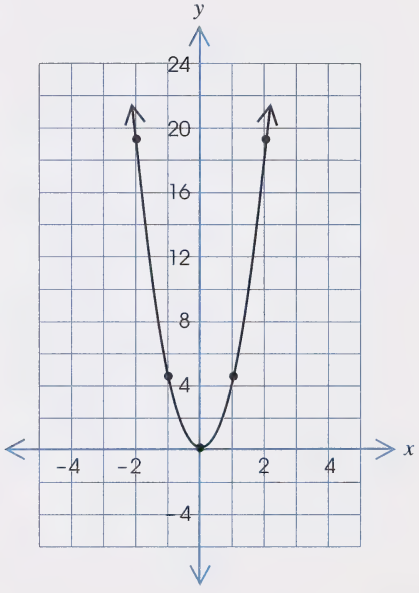
Time (s)	Distance (m)	
0	0	$=4.9(0)$
1	4.9	$=4.9(1)$
2	19.6	$=4.9(4)$
3	44.1	$=4.9(9)$
4	78.4	$=4.9(16)$

$\therefore\ d=4.9t^2$

d. $y=4.9x^2$

e. $y=4.9x^2$

x	y
0	0
1	4.9
−1	4.9
2	19.6
−2	19.6



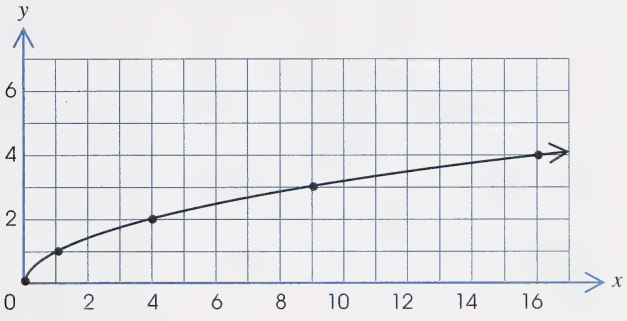
f. The graph in 9.a. shows only the positive values of x, whereas the graph in 9.e. shows both the positive and negative values of x.

g. The graph of $d = 4.9t^2$ shows only the right-hand half of $y = 4.9x^2$. This is because time is non-negative. Therefore, the domains differ.

	$d = 4.9t^2$	$y = 4.9x^2$
domain	$t \geq 0$	Reals
range	$d \geq 0$	$y \geq 0$

10. a. $y = \sqrt{x}$

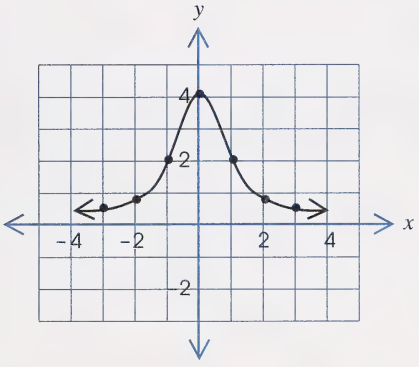
x	y
0	0
1	1
4	2
9	3
16	4



The range is $y \geq 0$.

b. $y = \dfrac{4}{x^2 + 1}$

x	y
0	4
1	2
−1	2
2	0.8
−2	0.8
3	0.4
−3	0.4



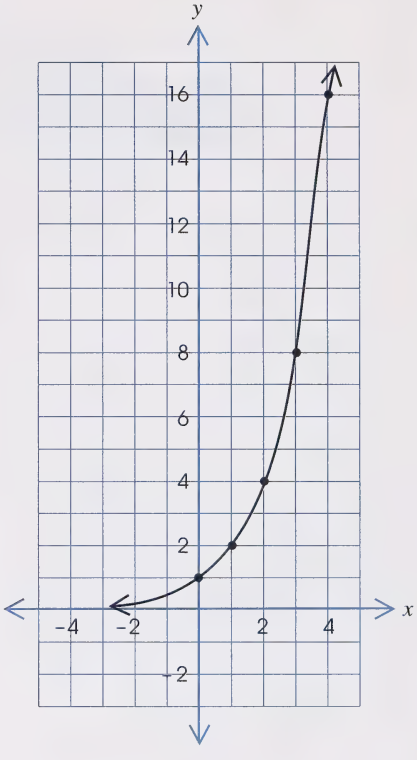
The range is $0 < y \leq 4$.

Section 2: Activity 3 (continued)

c. $y = 2^x$

x	y
0	1
1	2
2	4
3	8
4	16

The range is $y > 0$.



13. a. The domain is R because the graph may be extended infinitely to the left and right, and the range is $y \geq 0$.

b. The domain is R, and the range is $y \leq 4$.

c. The domain is $0 \leq x \leq 4$ because the radius of the circle is 2 (adding and subtracting 2 from the x-value of the centre), and the range is $0 \leq y \leq 4$.

d. The domain is $-7 \leq x \leq -1$ because the radius of the circle is 3 (adding and subtracting 3 from the x-value of the centre), and the range is $-6 \leq y \leq 0$.

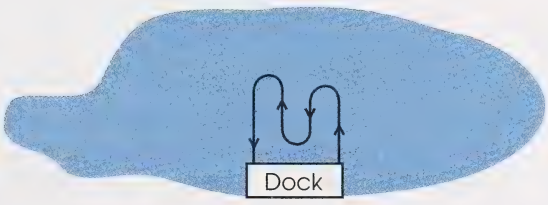
3. Textbook questions 1 to 5 of "Inquire," p. 228

1. McKoy led at the start of the race.
2. McKoy was passed by Thomson at the 6-s mark.
3. McKoy, who won the race, took the lead at the 80-m point, 30 m from the finish.
4. McKoy won the race with a time of 13.12 s.
5. Thomson's time was 14.8 s.

4. Textbook questions 1, 2, 3, 5, 11, and 12 of "Applications and Problem Solving," pp. 229 to 231

1. a. The water-skier skied away from the dock, then fell. After being picked up, the water-skier skied toward the dock, turned, skied away from the dock, and then fell a second time. This time, after being picked up, the skier returned to the dock.

b.

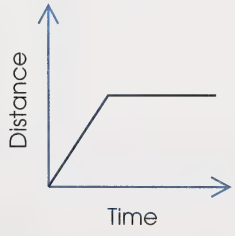


2. If the graphs represent distance travelled versus time, graphs (b) and (c) are possible. Both distance and time increase, as the graph is read from left to right.

Graphs (a) and (d) are impossible. For graph (a), time goes backwards! For graph (d), since the vertical axis represents total distance travelled, the graph's slope must be positive or, at least, 0. The last section has a negative slope. Distance must increase, not decrease.

3. For each graph, unlike question 2, the distance axis is the distance from the school, not the total distance walked.

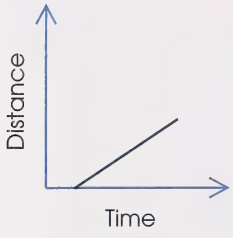
a.



The student started walking away from the school, then stopped to rest.

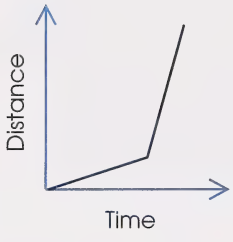
Section 2: Activity 3 (continued)

b.



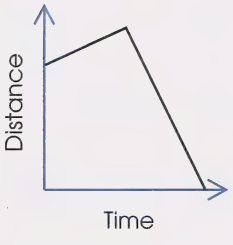
The student stayed at the school for awhile, then started walking away.

c.



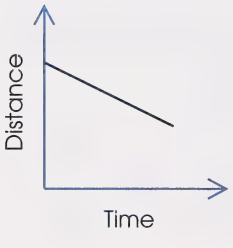
The student started walking away from the school, then began running away from the school.

d.



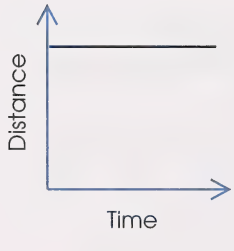
The student started walking from a point some distance from the school. He or she started walking away from the school at first, then turned and ran to the school.

e.



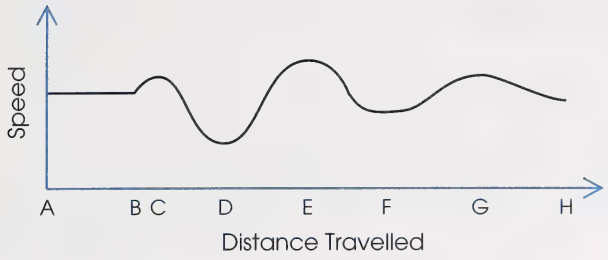
The student is walking towards the school, but still hasn't arrived.

f.



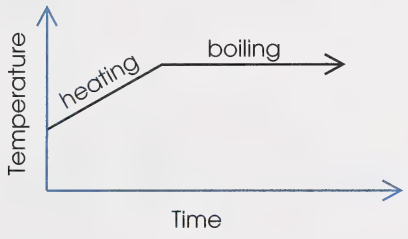
The student is at home, probably asleep in his or her bed.

5.

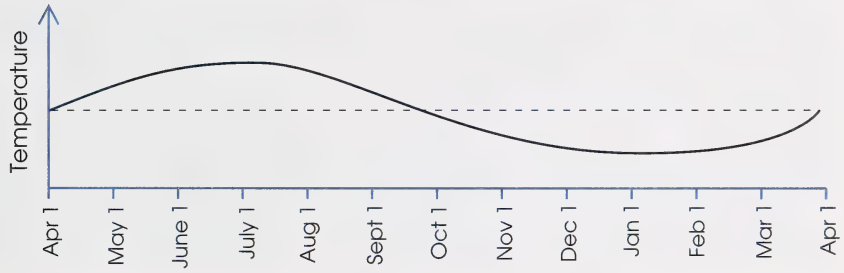


11. Answers may vary.

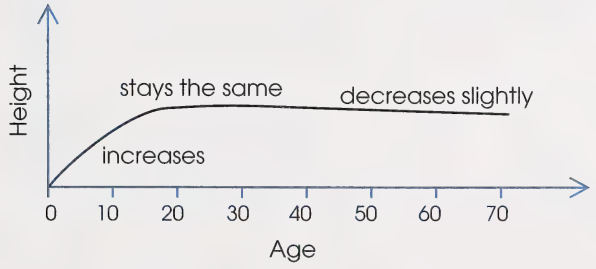
a.



b.



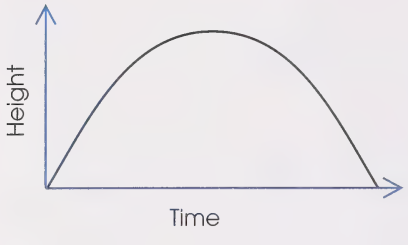
c.



Section 2: Activity 3 (continued)

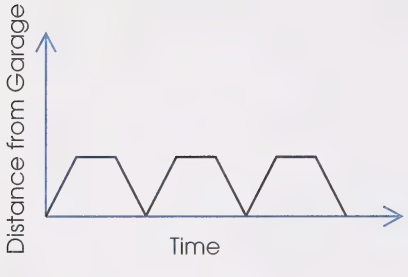
12. Answers may vary. Sample answers are given.

a.



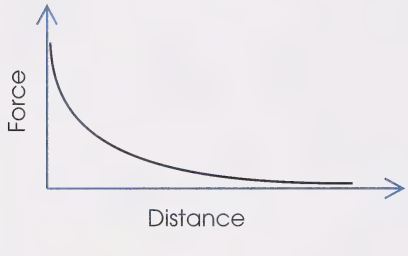
This graph could represent the height of a ball that was thrown straight up from ground level.

b.



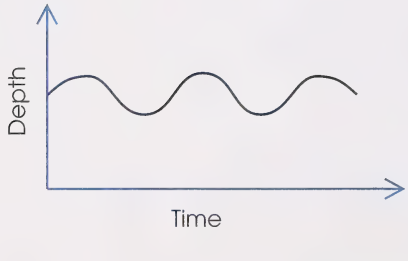
This graph could represent a student driver practising driving away from the garage, stopping, then driving towards the garage.

c.



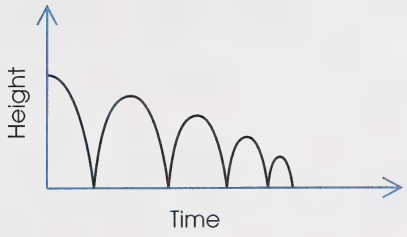
This graph could represent the force between two electrically charged particles as they separate.

d.



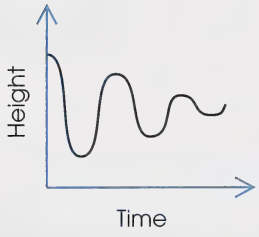
This graph could represent the depth of water in a harbour as the tides come in and go out.

e.



This graph could represent the height of a bouncing ball.

f.



This graph could represent the height of the tip of a ruler that is projecting over the top of a desk and vibrating after it was given a tug upward.

5. Textbook questions 1 to 5 of Investigation 1, "Polygons From 8 Squares," p. 210

1.

Polygon	Area (A)	Interior Points (I)	Perimeter (P)	Half the Perimeter (H)
1	8	2	14	7
2	8	3	12	6
3	8	1	16	8
4	8	3	12	6
5	8	0	18	9

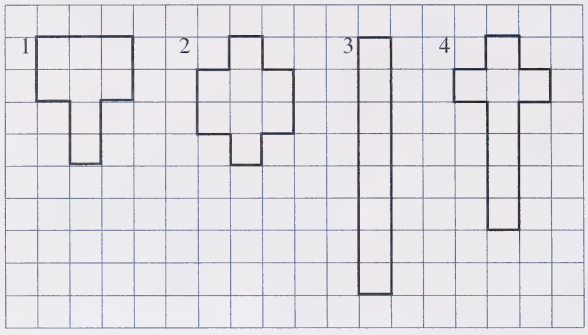
2. Half the perimeter, H, is equal to the area, A, minus the number of interior points, I, plus 1.

3. $H = A - I + 1$

4. $P = 2(A - I + 1)$

Section 2: Activity 3 (continued)

5. Answers will vary. A sample answer is given.



Polygon	Area (A)	Interior Points (I)	Perimeter (P)	Half the Perimeter (H)
1	8	2	14	7
2	8	2	14	7
3	8	0	18	9
4	8	0	18	9

These four polygons satisfy both expressions.

Section 2: Activity 4

1. a. Textbook question "Explore: Complete the Table," p. 232

Time (h)	Distance (km)
2	40
3	60
4	80
7	140
10	200

b. Textbook questions 1 to 3 of "Inquire," p. 232

1. $(2,40),\ (3,60),\ (4,80),\ (7,140),\ (10,200)$

2. No, distance is uniquely determined. For any given time, only one distance is possible.

3. Yes, this relation is a function. For each element of the domain, there is exactly one element in the range.

2. Textbook questions 1, 3, and 5 of "Practice," p. 234

1. Yes, it does represent a function. No two ordered pairs have the same x-coordinate.

3. No, it does not represent a function. The ordered pairs $(4,2)$ and $(4,6)$ have the same x-coordinate.

5. Yes, it does represent a function. No two ordered pairs have the same x-coordinate.

3. Textbook questions 7.a. to 7.c., 8.a. to 8.c., 9.a. to 9.c., and 10.a. to 10.c. of "Practice," p. 234

7. a. $f(x)=4x-5$

$f(2)=4(2)-5$

$=8-5$

$=3$

b. $f(x)=4x-5$

$f(7)=4(7)-5$

$=28-5$

$=23$

c. $f(x)=4x-5$

$f(0)=4(0)-5$

$=0-5$

$=-5$

8. a. $f(x)=8-2x$

$f(1)=8-2(1)$

$=8-2$

$=6$

b. $f(x)=8-2x$

$f(3)=8-2(3)$

$=8-6$

$=2$

c. $f(x)=8-2x$

$f(5)=8-2(5)$

$=8-10$

$=-2$

Section 2: Activity 4 (continued)

9. a. $g(x)=x^2+5$

$g(2)=(2)^2+5$

$=4+5$

$=9$

b. $g(x)=x^2+5$

$g(0)=(0)^2+5$

$=0+5$

$=5$

c. $g(x)=x^2+5$

$g(-2)=(-2)^2+5$

$=4+5$

$=9$

10. a. $f(n)=2n^2-4n+3$

$f(0)=2(0)^2-4(0)+3$

$=0-0+3$

$=3$

b. $f(n)=2n^2-4n+3$

$f(1)=2(1)^2-4(1)+3$

$=2-4+3$

$=1$

c. $f(n)=2n^2-4n+3$

$f(-2)=2(-2)^2-4(-2)+3$

$=2(4)+8+3$

$=8+8+3$

$=19$

4. Textbook questions 11, 12, and 14 of "Practice," p. 234

11. The domain is $\{2,1,0,-1,-2\}$.

The range is $\{5,2,1\}$.

Yes, this relation is a function. For each element in the domain, there is only one element of the range.

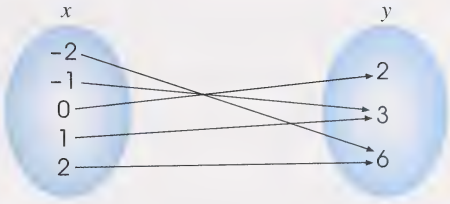
12. The domain is $\{3,2,1,0\}$.

The range is $\{1,0,-1,-2\}$.

No, this relation is not a function. The ordered pairs $(3,1)$ and $(3,-2)$ have the same x-coordinate.

14. a. $f(x) = x^2 + 2$, where $x = -2, -1, 0, 1, 2$

x	$f(x)$
-2	6
-1	3
0	2
1	3
2	6



b. The range is $\{2, 3, 3\}$.

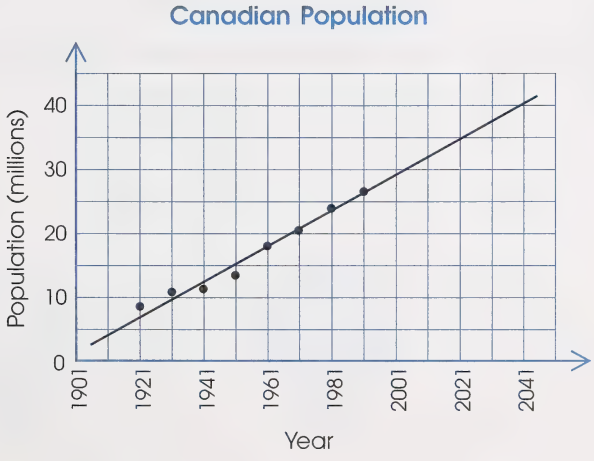
5. Textbook questions 17, 20, 22 to 26, and 31 of "Applications and Problem Solving," pp. 234 and 235

17. a.
$$f(s) = \frac{s}{1200}$$
$$f(2400) = \frac{2400}{1200}$$
$$= 2$$

$$f(s) = \frac{s}{1200}$$
$$f(3000) = \frac{3000}{1200}$$
$$= 2.5$$

b. Since the Mach number depends on speed, speed is the independent variable and the Mach number is the dependent variable.

20. a. Answers may vary. A sample answer is given.



Section 2: Activity 4 (continued)

b. The population in 1958 was about 17 million.

c. The population will reach 40 million around 2040.

22. Yes, this is a graph of a function; it passes the vertical line test.

The domain is $-5 \le x \le 5$.
The range is $0 \le y \le 6$.

23. No, this is not a graph of a function. Points $(2,1)$ and $(2,3)$ have the same x-coordinate.

The domain is $\{-3,-2,0,1,2,4.5\}$.
The range is $\{-2,-1,0,1,2,3\}$.

24. Yes, this is a graph of a function. For each x-value, there is exactly one y-value.

The domain is R.
The range is $y \ge -5$.

25. No, this is not a graph of a function. There are distinct ordered pairs with the same x-coordinate.

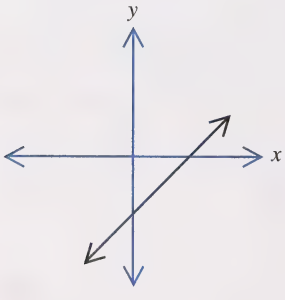
The domain is $-5 \le x \le 5$.
The range is $-5 \le y \le 5$.

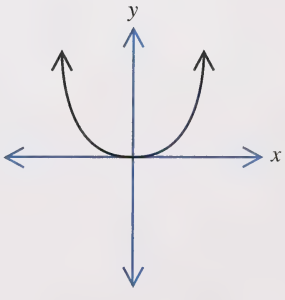
26. **a.** You can draw a vertical line through more than two points in graphs 23 and 25.

b. If a vertical line can be drawn through more than one point on the graph of a relation, the relation is not a function.

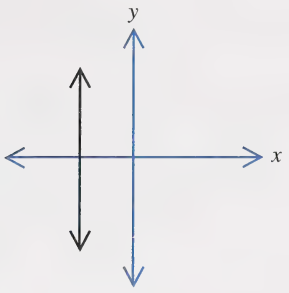
c. Answers may vary. A sample answer is given.

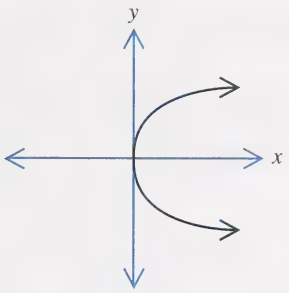
The following are examples of graphs of functions:





The following are examples of graphs of relations that are not functions:





31. a. An open dot means that the point is not included in the graph; a closed dot means that the point is included in the graph.

b. The graph is called a step function because it looks like a staircase.

c. The domain is $0 < x \leq 4$.
The range is $\{120, 200, 280, 360\}$.

d. Answers may vary. A sample answer is given.

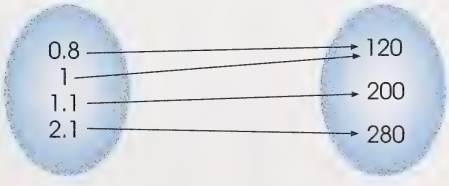
$(0.8, 120),\ (1, 120)$

e. Answers may vary. A sample answer is given.

$(1.1, 200),\ (2.1, 280)$

f. No, it is not possible to write two different ordered pairs with the same time value to represent points on the graph. It may appear that $(1, 120)$ and $(1, 200)$ are both on the graph; however, closer inspection reveals that there is a closed dot at $(1, 120)$ and an open dot at $(1, 200)$. This means that $(1, 120)$ is part of the graph and $(1, 200)$ is not.

g. Answers will vary. A sample answer is given.



Section 2: Activity 4 (continued)

6. Textbook questions 1 to 12 of "Mental Math: Subtracting Using Compatible Numbers," p. 209

1. $55-19=(55+1)-(19+1)$
$=56-20$
$=36$

2. $73-28=(73+2)-(28+2)$
$=75-30$
$=45$

3. $80-39=(80+1)-(39+1)$
$=81-40$
$=41$

4. $81-22=(81-2)-(22-2)$
$=79-20$
$=59$

5. $70-31=(70-1)-(31-1)$
$=69-30$
$=39$

6. $61-23=(61-3)-(23-3)$
$=58-20$
$=38$

7. Adjust the numbers so that you end up subtracting a whole number.

$6.4-2.9=(6.4+0.1)-(2.9+0.1)$
$=6.5-3.0$
$=3.5$

8. $10-3.8=(10+0.2)-(3.8+0.2)$
$=10.2-4.0$
$=6.2$

9. $7.2-4.3=(7.2-0.3)-(4.3-0.3)$
$=6.9-4.0$
$=2.9$

10. Adjust the numbers so that you end up subtracting by a number that is a multiple of 100.

$650-280=(650+20)-(280+20)$
$=670-300$
$=370$

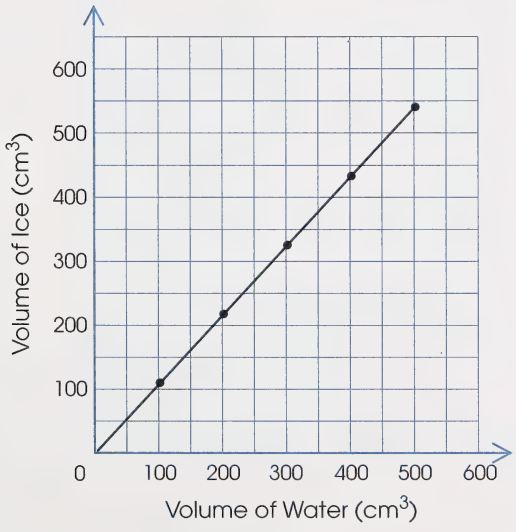
11. $1800-990=(1800+10)-(990+10)$
$=1810-1000$
$=810$

12. $710-320=(710-20)-(320-20)$
$=690-300$
$=390$

Section 2: Activity 5

1. a. Textbook questions a. and b. of "Explore: Draw a Graph," p. 236

a.



b. The function is linear.

Section 2: Activity 5 (continued)

b. Textbook questions 1 to 4 of "Inquire," p. 236

1.

Volume of Water (cm^3)	Volume of Ice (cm^3)	$\frac{\text{Volume of Ice}}{\text{Volume of Water}}$
100	109	$\frac{109}{100}=1.09$
200	218	$\frac{218}{200}=1.09$
300	327	$\frac{327}{300}=1.09$
400	436	$\frac{436}{400}=1.09$
500	545	$\frac{545}{500}=1.09$

The ratios are equal.

2. $I=1.09w$

3. a. $I=1.09w$

$=1.09(800)$

$=872$

The volume of ice formed is $872\ \text{cm}^3$.

b. $I=1.09w$

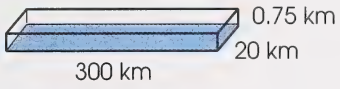
$w=\frac{I}{1.09}$

$=\frac{1308}{1.09}$

$=1200$

The volume of water needed is $1200\ \text{cm}^3$.

4. a. If the iceberg is a rectangular solid, it would look as follows:



$$\therefore V = \text{length} \times \text{width} \times \text{height}$$
$$= 300 \times 20 \times 0.75$$
$$= 4500$$

The volume of the iceberg is 4500 km^3.

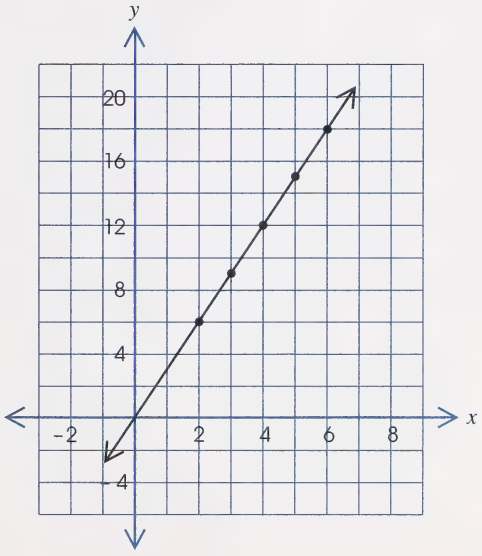
b. $$\text{Volume of water} = \frac{4500}{1.09}$$
$$\doteq 4128$$

When the iceberg melts, approximately 4128 km^3 of water is produced.

2. Textbook questions 1, 3, 5, 7, 9, and 11 of "Practice," pp. 238 and 239

1.

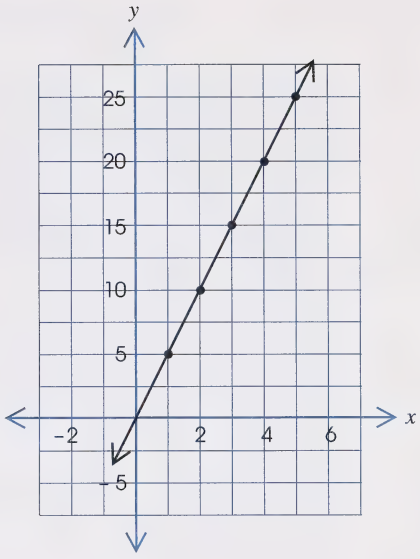
x	y
2	6
3	9
4	12
5	15
6	18



Section 2: Activity 5 (continued)

3.

x	y
1	5
2	10
3	15
4	20
5	25



5. a. $a \propto b$

$\therefore a = kb$

$\frac{a}{b} = k$

$\frac{10}{4} = k$

$k = 2.5$

b. $a = 2.5b$

7. a. $m \propto n$

$\therefore m = kn$

$\frac{m}{n} = k$

$\frac{11}{33} = k$

$k = \frac{1}{3}$

b. $m = \frac{1}{3}n$

9. Method 1

$c \propto d$

$\therefore\ c = kd$

$\frac{c}{d} = k$

$\frac{12}{-3} = k$

$k = -4$

$\therefore\ c = -4d$

When $d = -10$,

$c = -4(-10)$

$= 40$

Method 2

$\frac{12}{-3} = \frac{c}{-10}$

$-4 = \frac{c}{-10}$

$-4(-10) = \frac{c}{-10}(-10)$

$40 = c$

$c = 40$

11. Method 1

$r \propto s$

$\therefore\ r = ks$

$\frac{r}{s} = k$

$\frac{39}{26} = k$

$k = \frac{3}{2}$

$= 1.5$

$\therefore\ r = 1.5s$

When $r = 21$,

$21 = 1.5s$

$\frac{21}{1.5} = s$

$14 = s$

$s = 14$

Method 2

$\frac{39}{26} = \frac{21}{s}$

$1.5 = \frac{21}{s}$

$(1.5)s = \left(\frac{21}{s}\right)s$

$1.5s = 21$

$s = \frac{21}{1.5}$

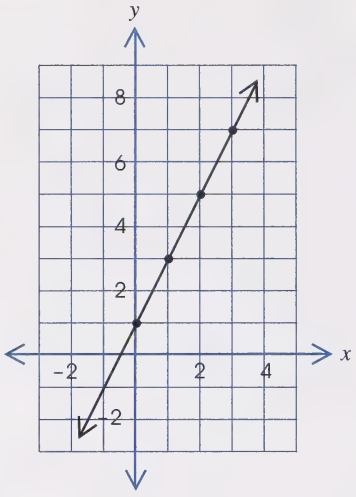
$= 14$

Section 2: Activity 5 (continued)

3. Textbook questions 13 and 14 of "Practice," p. 239

13. a. $y = 2x + 1$

x	y
0	1
1	3
2	5
3	7



b. This is a partial variation because the graph does not pass through the origin.

14. The graph is a partial variation. The equation of a partial variation is of the form $y = mx + b$.

Begin by finding m. Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (5, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7-4}{5-2}$$
$$= \frac{3}{3}$$
$$= 1$$

$\therefore\ y = 1x + b$

Next, find b by using either point given, since both points satisfy the equation. In this case, let $(x, y) = (2, 4)$.

$$y = x + b$$
$$4 = 2 + b$$
$$b = 2$$

Therefore, the equation that represents the function is $y = x + 2$.

4. Textbook questions 17 and 18 of "Applications and Problem Solving," p. 239

17. a. Let E represent the energy (in kJ) burned and t represent the time (in min).

$E \propto t$

$\therefore E = kt$

$200 = k(5)$

$\frac{200}{5} = k$

$k = 40$

$\therefore E = 40t$

b. $E = 40(3.5)$

$= 130 \text{ kJ}$

Paula burns 130 kJ of energy.

c. $320 = 40t$

$\frac{320}{40} = t$

$t = 8$

Paula would have to skip rope for 8 min.

18. a. Let b represent the height (in cm) the ball bounces and d represent the height (in cm) from which the ball is dropped.

$b \propto d$

$\therefore b = kd$

$15 = k(20)$

$\frac{15}{20} = k$

$k = \frac{3}{4}$

$\therefore b = \frac{3}{4}d$

Section 2: Activity 5 (continued)

b. $b = \frac{3}{4}(240)$

$= 180$

The height of the bounce is 180 cm.

c. $105 = \frac{3}{4}d$

$420 = 3d$

$d = 140$

The ball was dropped from a height of 140 cm.

5. Textbook questions a. and b. of "Logic Power," p. 240

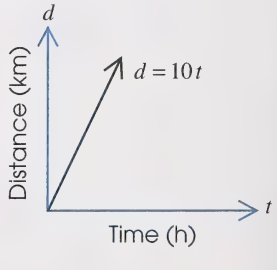
a. $\angle XYZ = 90°$

b. $\angle XYZ$ must be 90° because XY and YZ lie along two adjacent faces of the cube. Those faces are perpendicular; therefore, XY and YZ are perpendicular.

Section 2: Follow-up Activities

Extra Help

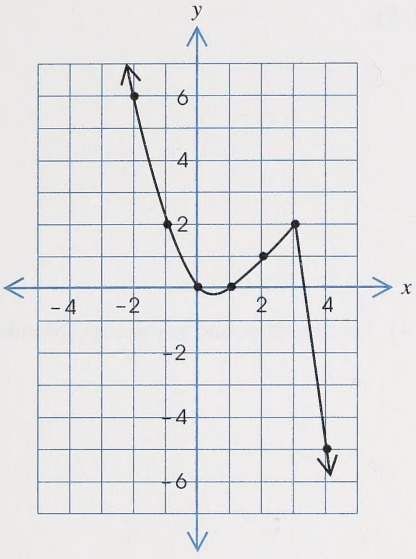
1. The graph is continuous since you can travel for any time period. The time can be any part of a minute or hour.



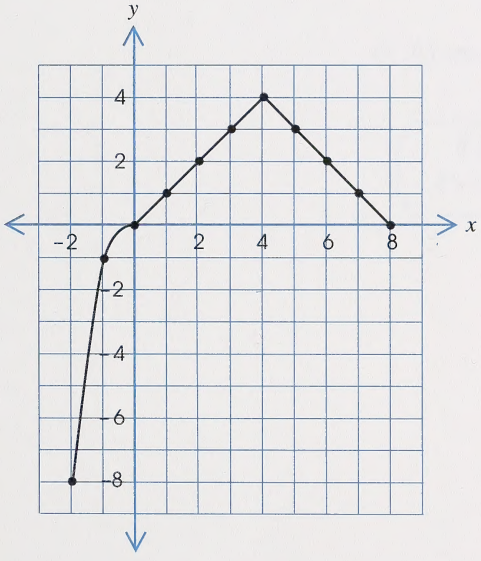
2. This graph is not continuous; it is a series of points. The points can not be joined because you can't purchase a fraction of a soft drink.

Enrichment

1. a.



b.



Enrichment (continued)

2. First, find the equation of the segment joining $(-4,1)$ and $(0,5)$.

Use the slope and y-intercept formula.

$y = mx + b$

$y = 1x + 5$ ($m = 1$ and $b = 5$)

$\therefore\ y = x + 5,\ \text{when}\ -4 \le x \le 0$

Next, find the equation of the segment joining $(0,5)$ and $(3,3)$. Use the slope and y-intercept formula.

$y = mx + b$

$y = -\frac{2}{3}x + 5$ ($m = -\frac{2}{3}$ and $b = 5$)

$\therefore\ y = -\frac{2}{3}x + 5,\ \text{when}\ 0 < x \le 3$

Lastly, find the equation of the segment joining $(3,3)$ and $(9,3)$.

This segment is horizontal.

$\therefore\ y = 3,\ \text{when}\ 3 < x < 9$

An open circle at $(9,3)$ indicates $x \neq 9$.

The piecewise function is defined as follows:

$y = x + 5,\ \text{when}\ -4 \le x \le 0$

$y = -\frac{2}{3}x + 5,\ \text{when}\ 0 < x \le 3$

$y = 3,\ \text{when}\ 3 < x < 9$

© 2002